LV. *An Attempt to explain ſome of the principal Phænomena of Electricity, by Means of an elaſtic Fluid: By the Honourable* Henry Cavendiſh, *F. R. S.*

Read Dec. 19, 1771, and Jan. 9, 1772.

SINCE I firſt wrote the following paper, I find that this way of accounting for the phænomena of electricity, is not new. Æpinus, in his Tentamen Theoriæ electricitatis & magnetiſmi, has made uſe of the ſame, or nearly the ſame hypotheſis that I have; and the concluſions he draws from it, agree nearly with mine, as far as he goes. However, as I have carried the theory much farther than he has done, and have conſidered the ſubject in a different, and, I flatter myſelf, in a more accurate manner, I hope the Society will not think this paper unworthy their acceptance.

The method I propoſe to follow is, firſt, to lay down the hypotheſis; next, to examine by ſtrict mathematical reaſoning, or at leaſt, as ſtrict reaſoning as the nature of the ſubject will admit of, what conſequences will flow from thence; and laſtly, to examine how far theſe conſequences agree with ſuch experiments as have yet been made on this ſubject. In a future paper, I intend to give the reſult of ſome experiments I am making, with intent to examine ſtill further the truth of this hypotheſis, and to find out the law of the electric attraction and repulſion.

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Fig. 1.

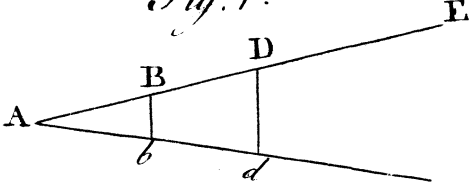
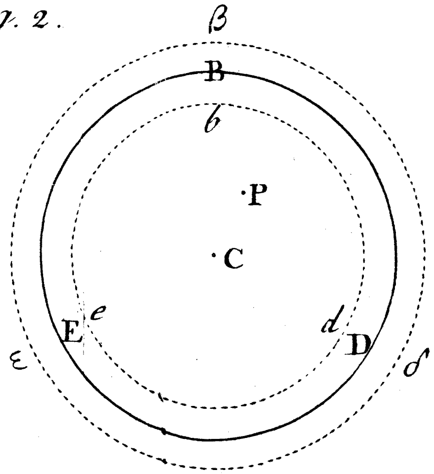


Fig. 2.



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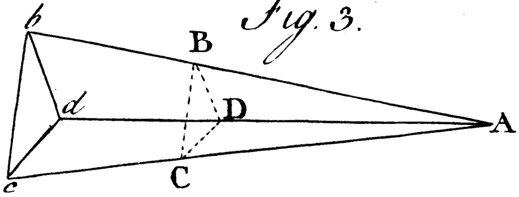


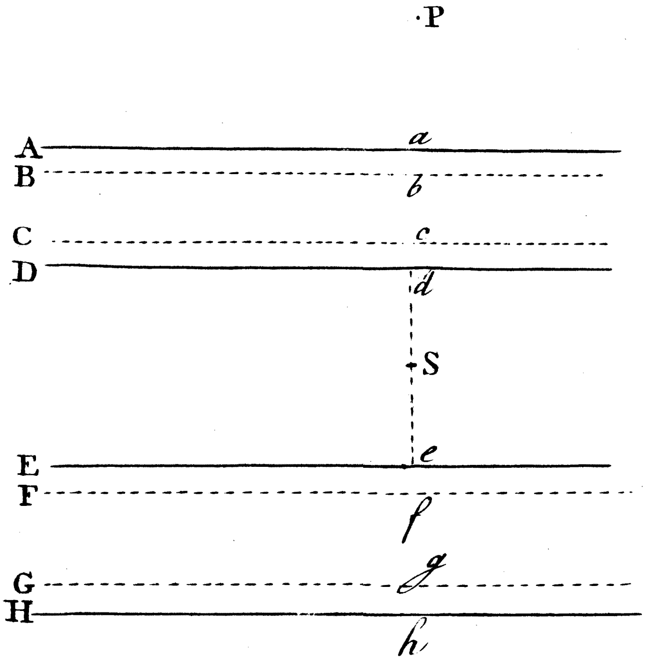
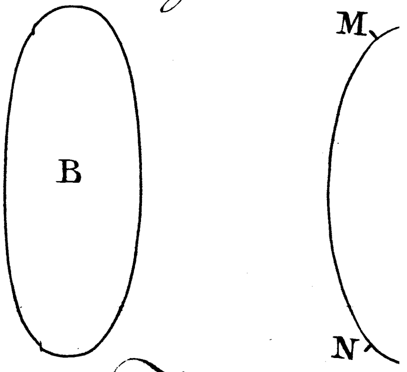


Fig. 4.

Fig. 5.



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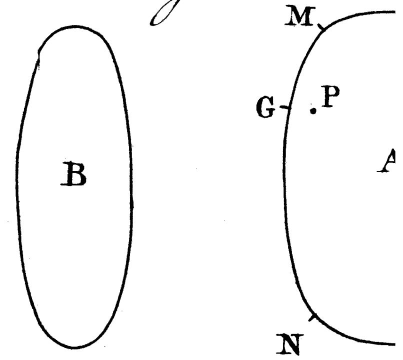

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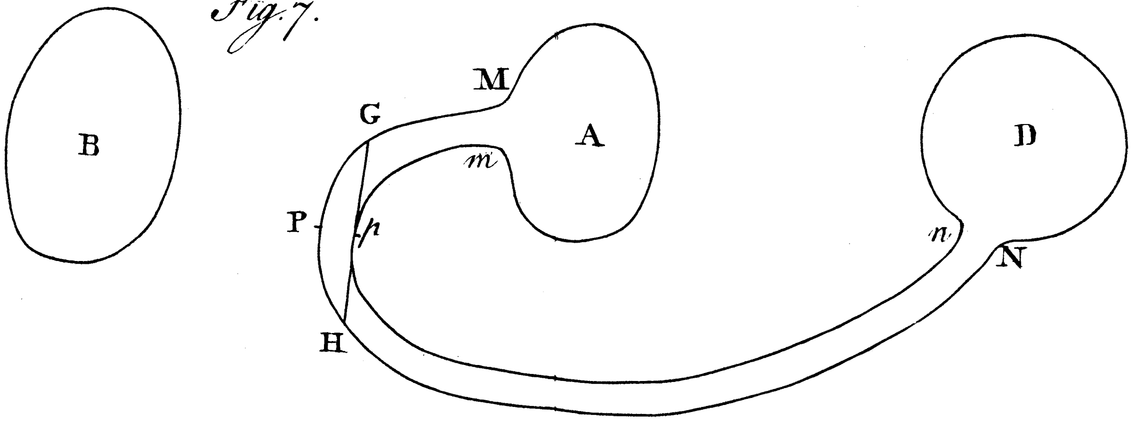

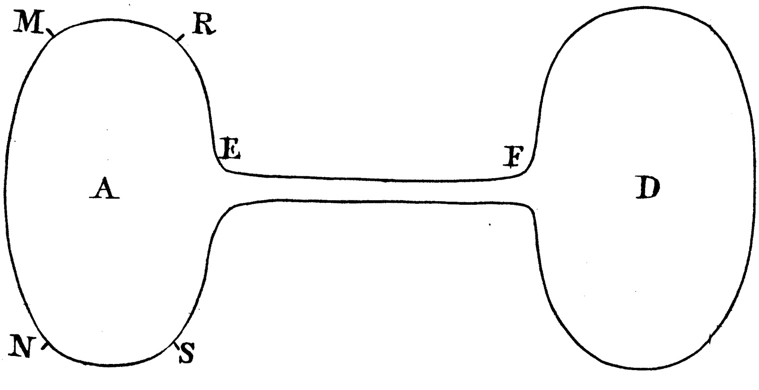

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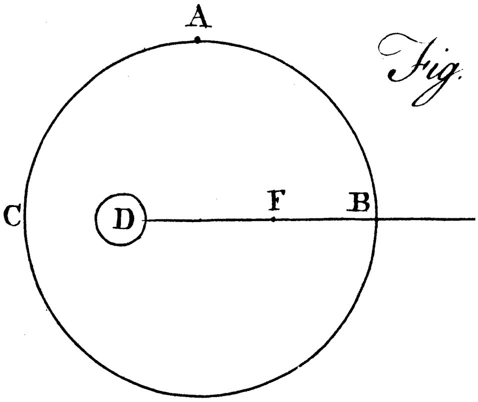
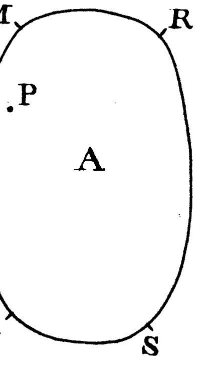




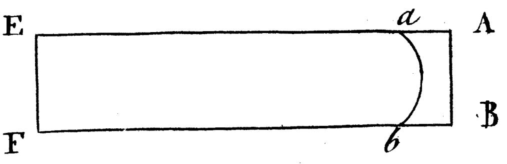
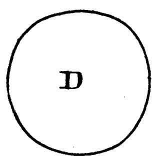


Fig. 1.



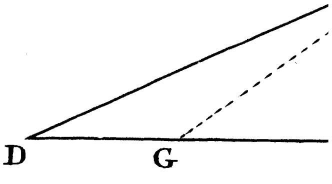
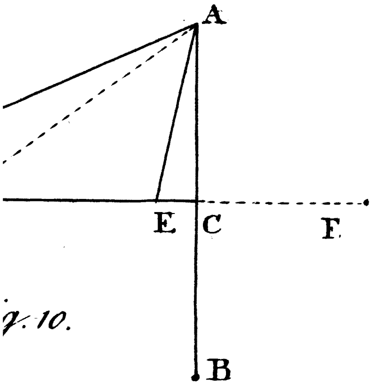


Fig. 10.

Fig. 8.



g. 10.

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HYPOTHESIS.

THERE is a ſubſtance, which I call the electric fluid, the particles of which repel each other and attract the particles of all other matter, with a force inverſely as ſome leſs power of the diſtance than the cube: the particles of all other matter alſo, repel each other, and attract thoſe of the electric fluid, with a force varying according to the ſame power of the diſtances. Or, to expreſs it more conciſely, if you look upon the electric fluid as matter of a contrary kind to other matter, the particles of all matter, both thoſe of the electric fluid and of other matter, repel particles of the ſame kind, and attract thoſe of a contrary kind, with a force inverſely as ſome leſs power of the diſtance than the cube.

For the future, I would be underſtood never to comprehend the electric fluid under the word matter, but only ſome other ſort of matter.

It is indifferent whether you ſuppoſe all ſorts of matter to be indued in an equal degree with the foregoing attraction and repulſion, or whether you ſuppoſe ſome ſorts to be indued with it in a greater degree than others; but it is likely that the electric fluid is indued with this property in a much greater degree than other matter; for in all probability the weight of the electric fluid in any body bears but a very ſmall proportion to the weight of the matter; but yet the force with which the electric fluid therein attracts any particle of matter muſt be equal to the force with which the matter therein repels that particle; otherwiſe the body would appear electrical, as will be ſhewn hereafter.

To explain this hypotheſis more fully, ſuppoſe that 1 grain of electric fluid attracts a particle of matter, at a given diſtance, with as much force as *n* grains of any matter, lead for inſtance, repel it: then will 1 grain of electric fluid repel a particle of electric fluid with as much force as *n* grains of lead attract it; and 1 grain of electric fluid will repel 1 grain of electric fluid with as much force as *n* grains of lead repel *n* grains of lead.

All bodies in their natural ſtate, with regard to electricity, contain ſuch a quantity of electric fluid interſperſed between their particles, that the attraction of the electric fluid in any ſmall part of the body on a given particle of matter ſhall be equal to the repulſion of the matter in the ſame ſmall part on the ſame particle. A body in this ſtate I call ſaturated with electric fluid: if the body contains more than this quantity of electric fluid, I call it overcharged: if leſs, I call it undercharged. This is the hypotheſis; I now proceed to examine the conſequences which will flow from it.

LEMMA I.

Let EA*e* (TAB. XX. fig. 1.) repreſent a cone continued infinitely; let A be the vertex, and B*b* and D*d* planes parallel to the baſe; and let the cone be filled with uniform matter, whoſe particles repel each other with a force inverſely as the *n* power of the diſtance. If *n* is greater than 3, the force with which a particle at A is repelled by E B *be* or all that part of the cone beyond B *b* is as $\frac{1}{AB^{n-3}}$.

For ſuppoſing A B to flow, the fluxion of EB*be* is proportional to $-\dot{AB} \times AB^{2}$, and the fluxion of its repulſion on A is proportional to $\frac{-\dot{AB}}{AB^{n-2}}$; the fluent of which is $\frac{1}{n-3 \times AB^{n-3}}$; which when AB is infinite is equal to nothing; conſequently the repulſion of EB*be* is proportional to $\frac{1}{n-3 \times AB^{n-3}}$ or to $\frac{1}{AB^{n-3}}$.

COROLLARY.

If AB is infinitely ſmall, $\frac{1}{AB^{n-3}}$ is infinitely great; therefore the repulſion of that part of the cone between A and B*b*, on A, is infinitely greater than the repulſion of all that beyond it.

LEMMA II.

By the ſame method of reaſoning it appears, that if *n* is equal to 3, the repulſion of the matter between B *b* and D *d* on a particle at A, is proportional to the logarithm of $\frac{AD}{AB}$; conſequently, the repulſion of that part is infinitely ſmall in reſpect of that between A and B*b*, and alſo infinitely ſmall in reſpect of that beyond D *d*.

LEMMA III.

In like manner, if n is leſs than 3, the repulſion of the part between A and B*b* on A is proportional to AB^{3-n}: conſequently the repulſion of the matter between A and B*b* on A, is infinitely ſmall in reſpect of that beyond it.

COROLLARY.

It is eaſy to ſee from theſe three lemmata, that, if the electric attraction and repulſion had been ſuppoſed to be inverſely, as ſome higher power of the diſtance than the cube; a particle could not have been ſenſibly affected by the repulſion of any fluid, except what was placed cloſe to it. If the repulſion was inverſely, as the cube of the diſtance, a particle could not be ſenſibly affected by the repulſion of any finite quantity of fluid, except what was cloſe to it. But as the repulſion is ſuppoſed to be inverſely as ſome power of the diſtance leſs than the cube, a particle may be ſenſibly affected by the repulſion of a finite quantity of fluid, placed at any finite diſtance from it.

DEFINITION.

If the electric fluid in any body, is by any means confined in ſuch manner that it cannot move from one part of the body to the other; I call it immoveable: if it is able to move readily from one part to another, I call it moveable.

PROPOSITION I.

A body overcharged with electric fluid attracts or repels a particle of matter or fluid, and is attracted or repelled by it, with exactly the ſame force as it would, if the matter in it, together with ſo much of the fluid as is ſufficient to ſaturate it, was taken away, or as if the body conſiſted only of the redundant fluid in it. In like manner an undercharged body attracts or repels with the ſame force, as if it conſiſted only of the redundant matter; the electric fluid, together with ſo much of the matter as is ſufficient to ſaturate it, being taken away.

This is evident from the definition of ſaturation.

PROP. II.

Two over or undercharged bodies attract or repel each other with juſt the ſame force that they would, if each body conſiſted only of the redundant fluid in it, if overcharged, or of the redundant matter in it, if undercharged.

For, let the two bodies be called A and B; by the laſt propoſition the redundant ſubſtance in B impels each particle of fluid and matter in A, and conſequently impels the whole body A, with the ſame force that the whole body B impels it: for the ſame reaſon the redundant ſubſtance in A impels the redundant ſubſtance in B, with the ſame force that the whole body A impels it. It is ſhewn therefore, that the whole body B impels the whole body A, with the ſame force that the redundant ſubſtance in B impels the whole body A, or with which the whole body A impels the redundant ſubſtance in B; and that the whole body A impels the redundant ſubſtance in B, with the ſame force that the redundant ſubſtance in A impels the redundant ſubſtance in B; therefore the whole body B impels the whole body A, with the ſame force with which the redundant ſubſtance in A impels the redundant ſubſtance in B, or with which the redundant ſubſtance in B impels the redundant ſubſtance in A.

COROLLARY.

Let the matter in all the reſt of ſpace, except in two given bodies, be ſaturated with immoveable fluid; and let the fluid in thoſe two bodies be alſo immoveable. Then, if one of the bodies is ſaturated, and the other either over or undercharged, they will not at all attract or repel each other.

If the bodies are both overcharged, they will repel each other.

If they are both undercharged, they will alſo repel each other.

If one is overcharged and the other undercharged, they will attract each other.

N. B. In this corollary, when I call a body overcharged, I would be underſtood to mean, that it is overcharged in all parts, or at leaſt no where undercharged: in like manner, when I call it undercharged, I mean that it is undercharged in all parts, or at leaſt no where overcharged.

P R O P. III.

If all the bodies in the univerſe are ſaturated with electric fluid, it is plain that no part of the fluid can have any tendency to move.

P R O P. IV.

If the quantity of electric fluid in the univerſe is exactly ſufficient to ſaturate the matter therein, but unequally diſperſed, ſo that ſome bodies are overcharged and others undercharged; then, if the electric fluid is not confined, it will immediately move till all the bodies in the univerſe are ſaturated.

For, ſuppoſing that any body is overcharged, and the bodies near it are not, a particle at the ſurface of that body will be repelled from it by the redundant fluid within; conſequently ſome fluid will run out of that body; but if the body is undercharged, a particle at its ſurface will be attracted towards the body by the redundant matter within, ſo that ſome fluid will run into the body.

N. B. In Prob. IV. Caſe III. there will be ſhewn an exception to this propoſition; there may perhaps be ſome other exceptions to it: but I think there can be no doubt, but what this propoſition muſt hold good in general.

LEMMA IV.

Let BDE, bde, and $\beta\delta\varepsilon$ (fig. 2.) be concentric ſpherical ſurfaces, whoſe center is C: if the ſpace * Bb is filled with uniform matter, whoſe particles repel with a force inverſely, as the ſquare of the diſtance, a particle placed any where within the ſpace Cb, as at P, will be repelled with as much force in one direction as another, or it will not be impelled in any direction. This is demonſtrated in Newt. Princip. liber I. prop. lxx. It follows alſo from his demonſtration, that if the repulſion is inverſely, as ſome higher power of the diſtance than the ſquare, the particle P will be impelled towards the center; and if the repulſion is inverſely as ſome lower power than the ſquare, it will be impelled from the center.

LEMMA V.

If the repulſion is inverſely as the ſquare of the diſtance, a particle placed any where without the ſphere BDE, is repelled by that ſphere, and alſo by the ſpace Bb, with the ſame force that it would if all the matter therein was collected in the center of the

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* By the ſpace Bb or Bβ, I mean the ſpace comprehended between the ſpherical ſurfaces BDE and bde, or between BDE and $\beta\delta\varepsilon$: by the ſpace Cb or Cβ, I mean the ſpheres bde or $\beta\delta\varepsilon$.

ſphere; provided the denſity of the matter therein is every where the ſame at the ſame diſtance from the center. This is eaſily deduced from prop. 71. of the ſame book, and has been demonſtrated by other authors.

PROP. V.

PROBLEM I. Let the ſphere BDE be filled with uniform ſolid matter, overcharged with electric fluid: let the fluid therein be moveable, but unable to eſcape from it: let the fluid in the reſt of infinite ſpace be moveable, and ſufficient to ſaturate the matter therein; and let the matter in the whole of infinite ſpace, or at leaſt in the ſpace Bβ, whoſe dimenſions will be given below, be uniform and ſolid; and let the law of the electric attraction and repulſion be inverſely as the ſquare of the diſtance: it is required to determine in what manner the fluid will be diſpoſed both within and without the globe.

Take the ſpace B*b* ſuch, that the interſtices between the particles of matter therein ſhall be juſt ſufficient to hold a quantity of electric fluid, whoſe particles are preſſed cloſe together, ſo as to touch each other, equal to the whole redundant fluid in the globe, beſides the quantity requiſite to ſaturate the matter in B*b*; and take the ſpace Bβ ſuch, that the matter therein ſhall be juſt able to ſaturate the redundant fluid in the globe: then, in all parts of the ſpace B*b*, the fluid will be preſſed cloſe together, ſo

that

that its particles ſhall touch each other; the ſpace Bβ will be intirely deprived of fluid; and in the ſpace C*b*, and all the reſt of infinite ſpace, the matter will be exactly ſaturated.

For, if the fluid is diſpoſed in the above-mentioned manner, a particle of fluid placed anywhere within the ſpace C*b* will not be impelled in any direction by the fluid in B*b*, or the matter in Bβ, and will therefore have no tendency to move: a particle placed anywhere without the ſphere $\beta\delta\epsilon$ will be attracted with juſt as much force by the matter in Bβ, as it is repelled by the redundant fluid in B*b*, and will therefore have no tendency to move: a particle placed anywhere within the ſpace B*b*, will indeed be repelled towards the ſurface, by all the redundant fluid in that ſpace which is placed nearer the center than itſelf; but as the fluid in that ſpace is already preſſed as cloſe together as poſſible, it will not have any tendency to move; and in the ſpace Bβ there is no fluid to move, ſo that no part of the fluid can have any tendency to move.

Moreover, it ſeems impoſſible for the fluid to be at reſt, if it is diſpoſed in any other form; for if the denſity of the fluid is not everywhere the ſame at the ſame diſtance from the center, but is greater near *b* than near *d*, a particle placed anywhere between thoſe two points will move from *b* towards *d*; but if the denſity is everywhere the ſame at the ſame diſtance from the center, and the fluid in B*b* is not preſſed cloſe together, the ſpace C*b* will be overcharged, and conſequently a particle at *b* will be repelled from the center, and cannot be at reſt: in like manner, if there is any fluid in Bβ, it cannot be at reſt: and, by the ſame kind of reaſoning, it might be ſhewn, that, if the fluid is not ſpread uniformly within the ſpace C *b*, and without the ſphere $\beta\delta\epsilon$, it cannot be at reſt.

COROLLARY I.

If the globe BDE is undercharged, every thing elſe being the ſame as before, there will be a ſpace B *b*, in which the matter will be intirely deprived of fluid, and a ſpace B β, in which the fluid will be preſſed cloſe together; the matter in B *b* being equal to the whole redundant matter in the globe, and the redundant fluid in B β, being juſt ſufficient to ſaturate the matter in B *b*: and in all the reſt of ſpace the matter will be exactly ſaturated. The demonſtration is exactly ſimilar to the foregoing.

COROL. II.

The fluid in the globe BDE will be diſpoſed in exactly the ſame manner, whether the fluid without is immoveable, and diſpoſed in ſuch manner, that the matter ſhall be everywhere ſaturated, or whether it is diſpoſed as above deſcribed; and the fluid without the globe will be diſpoſed in juſt the ſame manner, whether the fluid within is diſpoſed uniformly, or whether it is diſpoſed as above deſcribed.

PROP. VI.

Prob. 2. To determine in what manner the fluid will be diſpoſed in the globe BDE, ſuppoſing every thing as in the laſt problem, except that the fluid on the outſide of the globe is immoveable, and diſpoſed in ſuch manner as everywhere to ſaturate the matter, and that the electric attraction and repulſion is inverſely, as ſome other power of the diſtance than the ſquare.

I am not able to anſwer this problem accurately; but I think we may be certain of the following circumſtances.

Case 1. Let the repulſion be inverſely as ſome power of the diſtance between the ſquare and the cube, and let the globe be overcharged.

It is certain that the denſity of the fluid will be everywhere the ſame, at the ſame diſtance from the center. Therefore, firſt, There can be no ſpace as C*b*, within which the matter will be everywhere ſaturated; for a particle at *b* is impelled towards the center, by the redundant fluid in B*b*, and will therefore move towards the center, unleſs C*b* is ſufficiently overcharged to prevent it. Secondly, The fluid cloſe to the ſurface of the ſphere will be preſſed cloſe together; for otherwiſe a particle ſo near to it, that the quantity of fluid between it and the ſurface ſhould be very ſmall, would move towards it; as the repulſion of the ſmall quantity of fluid between it and the ſurface, would be unable to balance the repulſion of the fluid on the other ſide. Whence, I think, we may conclude, that the denſity of the fluid will increaſe gradually from the center to the ſurface, where the particles will be preſſed cloſe together: whether the matter exactly at the center will be overcharged, or only ſaturated, I cannot tell.

COROLLARY.

For the ſame reaſon, if the globe be undercharged, I think we may conclude, that the denſity of the fluid will diminiſh gradually from the center to the ſurface, where the matter will be entirely deprived of fluid.

CASE 2. Let the repulſion be inverſely as ſome power of the diſtance leſs than the ſquare; and let the globe be overcharged.

There will be a ſpace B *b*, in which the particles of the fluid will be everywhere preſſed cloſe together; and the quantity of redundant fluid in that ſpace will be greater than the quantity of redundant fluid in the whole globe BDE; ſo that the ſpace C *b*, taken all together, will be undercharged: but I cannot tell in what manner the fluid will be diſpoſed in that ſpace.

For it is certain, that the denſity of the fluid will be everywhere the ſame, at the ſame diſtance from the center. Therefore, let *b* be any point where the fluid is not preſſed cloſe together, then will a particle at *b* be impelled towards the ſurface, by the redundant fluid in the ſpace B*b*; therefore, unleſs the ſpace C*b* is undercharged, the particle will move towards the ſurface.

COROLLARY.

For the ſame reaſon, if the globe is undercharged, there will be a ſpace B*b*, in which the matter will be intirely deprived of fluid, the quantity of matter therein being more than the whole redundant matter in the globe; and, conſequently, the ſpace C*b*, taken all together, will be overcharged.

LEMMA VI.

Let the whole ſpace comprehended between two parallel planes, infinitely extended each way, be filled with uniform matter, the repulſion of whoſe particles is inverſely as the ſquare of the diſtance; the plate of matter formed thereby will repel a particle of matter with exactly the ſame force, at whatever diſtance from it, it be placed.

For, ſuppoſe that there are two ſuch plates, of equal thickneſs, placed parallel to each other, let A (fig. 3.) be any point not placed in or between the two plates: let BCD repreſent any part of the neareſt plate: draw the lines AB, AC, and AD, cutting the furtheſt plate in *b*, *c*, and *d*; for it is plain, that if they cut one plate, they muſt, if produced, cut the other: the triangle BCD is to the triangle *bcd*, as AB^2 to Ab^2; therefore a particle of matter at A will be repelled with the ſame force by the matter in the triangle BCD, as by that in *bcd*. Whence it appears, that a particle at A will be repelled with as much force by the neareſt plate, as by the more diſtant; and conſequently, will be impelled with the ſame force by either plate, at whatever diſtance from it it be placed.

COROLLARY.

If the repulſion of the particles is inverſely as ſome higher power of the diſtance than the ſquare, the plate will repel a particle with more force, if its diſtance be ſmall than if it be great; and if the repulſion is inverſely as ſome lower power than the ſquare, it will repel a particle with leſs force, if its diſtance be ſmall than if it be great.

PROP. VII.

PROB. 3. In fig. 4. let the parallel lines A*a*, B*b*, &c. repreſent parallel planes infinitely extended each way: let the ſpaces * AD and EH be filled with uniform ſolid matter: let the electric fluid in each of thoſe ſpaces be moveable and unable to eſcape: and let all the reſt of the matter in the univerſe be ſaturated with immoveable fluid; and let the electric attraction and repulſion be inverſely as the ſquare of the diſtance. It is required to determine in what manner the fluid will be diſpoſed in the ſpaces AD and EH, according as one or both of them are over or undercharged.

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* By the ſpace AD or AB, &c. I mean the ſpace comprehended between the planes A*a* and D*d*, or between A*a* and B*b*.

Let A D be that ſpace which contains the greateſt quantity of redundant fluid, if both ſpaces are overcharged, or which contains the leaſt redundant matter, if both are undercharged; or, if one is overcharged, and the other undercharged, let A D be the overcharged one. Then, firſt, There will be two ſpaces, A B and G H, which will either be intirely deprived of fluid, or in which the particles will be preſſed cloſe together; namely, if the whole quantity of fluid in A D and E H together, is leſs than ſufficient to ſaturate the matter therein, they will be intirely deprived of fluid; the quantity of redundant matter in each being half the whole redundant matter in A D and E H together: but if the fluid in A D and E H together is more than ſufficient to ſaturate the matter, the fluid in A B and G H will be preſſed cloſe together; the quantity of redundant fluid in each being half the whole redundant fluid in both ſpaces. 2dly, In the ſpace C D the fluid will be preſſed cloſe together; the quantity of fluid therein being ſuch, as to leave juſt enough fluid in B C to ſaturate the matter therein. 3dly, The ſpace E F will be intirely deprived of fluid; the quantity of matter therein being ſuch, that the fluid in F G ſhall be juſt ſufficient to ſaturate the matter therein: conſequently, the redundant fluid in C D will be juſt ſufficient to ſaturate the redundant matter in E F; for as A B and G H together contain the whole redundant fluid or matter in both ſpaces, the ſpaces B D and E G together contain their natural quantity of fluid; and therefore, as B C and F G each contain their natural quantity of fluid, the ſpaces C D and E F together contain their natural quantity of fluid. And, 4thly, The ſpaces B C and F G will be ſaturated in all parts.

For, firſt, If the fluid is diſpoſed in this manner, no particle of it can have any tendency to move: for a particle placed anywhere in the ſpaces B C and F G, is attracted with juſt as much force by E F, as it is repelled by C D; and it is repelled or attracted with juſt as much force by A B, as it is in a contrary direction by G H, and, conſequently, has no tendency to move. A particle placed anywhere in the ſpace C D, or in the ſpaces A B and G H, if they are overcharged, is indeed repelled with more force towards the planes D *d*, A *a*, and H *b*, than it is in the contrary direction; but as the fluid in thoſe ſpaces is already as much compreſſed as poſſible, the particle will have no tendency to move.

2dly, It ſeems impoſſible that the fluid ſhould be at reſt, if it is diſpoſed in any other manner: but as this part of the demonſtration is exactly ſimilar to the latter part of that of Problem the firſt, I ſhall omit it.

COROL. I.

If the two ſpaces A D and E H are both overcharged, the redundant fluid in C D is half the difference of the redundant fluid in thoſe ſpaces: for half the difference of the redundant fluid in thoſe ſpaces, added to the quantity in A B, which is half the ſum, is equal to the whole quantity in A D. For a like reaſon, if A D and E H are both undercharged, the redundant matter in E F is half the difference of the redundant matter in thoſe ſpaces; and if A D is overcharged, and E H undercharged, the redundant fluid in CD exceeds half the redundant fluid in AD, by a quantity ſufficient to ſaturate half the redundant matter in E H.

COROL. II.

It was before ſaid, that the fluid in the ſpaces A B and G H (when there is any fluid in them) is repelled againſt the planes A *a* and H *b*; and, conſequently, would run out through thoſe planes, if there was any opening for it to do ſo. The force with which the fluid preſſes againſt the planes A*a* and H*b*, is that with which the redundant fluid in A B is repelled by that in G H; that is, with which half the redundant fluid in both ſpaces is repelled by an equal quantity of fluid. Therefore, the preſſure againſt A *a* and H *b* depends only on the quantity of redundant fluid in both ſpaces together, and not at all on the thickneſs or diſtance of thoſe ſpaces, or on the proportion in which the fluid is divided between the two ſpaces. If there is no fluid in A B and G H, a particle placed on the outſide of the ſpaces A D and E H, contiguous to the planes A *a* or H *b*, is attracted towards thoſe planes by all the matter in A B and G H, *id eſt*, by all the redundant matter in both ſpaces; and, conſequently, endeavours to inſinuate itſelf into the ſpace A D or E H; and the force with which it does ſo, depends only on the quantity of redundant matter in both ſpaces together. The fluid in C D alſo preſſes againſt the plane D *d*, and the force with which it does ſo, is that with which the redundant fluid in C D is attracted by the matter in E F.

Corol. III.

If A D is overcharged, and E H undercharged, and the redundant fluid in A D is exactly ſufficient to ſaturate the redundant matter in E H, all the redundant fluid in A D will be collected in the ſpace C D, where it will be preſſed cloſe together: the ſpace E F will be intirely deprived of fluid, the quantity of matter therein being juſt ſufficient to ſaturate the redundant fluid in C D, and the ſpaces A C and F H will be everywhere ſaturated. Moreover, if an opening is made in the planes A *a* or H *b*, the fluid within the ſpaces A D or E H will have no tendency to run out thereat, nor will the fluid on the outſide have any tendency to run in at it: a particle of fluid too placed anywhere on the outſide of both ſpaces, as at P, will not be at all attracted or repelled by thoſe ſpaces, any more than if they were both ſaturated; but a particle placed anywhere between thoſe ſpaces, as at S, will be repelled from *d* towards *e*; and if a communication was made between the two ſpaces, by the canal *d e*, the fluid would run out of A D into E H, till they were both ſaturated.

PROP. VIII.

PROB. 4. To determine in what manner the fluid will be diſpoſed in the ſpace AD, ſuppoſing that all the reſt of the univerſe is ſaturated with immoveable fluid, and that the electric attraction and repulſion is inverſely as ſome other power of the diſtance than the ſquare.

I am not able to anſwer this Problem accurately, except when the repulſion is inverſely as the ſimple or ſome lower power of the diſtance; but I think we may be certain of the following circumſtances.

CASE 1. Let the repulſion be inverſely as ſome power of the diſtance between the ſquare and the cube, and let AD be overcharged.

Firſt, It is certain that the denſity of the fluid muſt be everywhere the ſame, at the ſame diſtance from the planes A*a* and D*d*. 2dly, There can be no ſpace as BC, of any ſenſible breadth, in which the matter will not be overcharged. And, 3dly, The fluid cloſe to the planes A*a* and D*d* will be preſſed cloſe together. Whence, I think, we may conclude, that the denſity of the fluid will increaſe gradually from the middle of the ſpace to the outſide, where it will be preſſed cloſe together. Whether the matter exactly in the middle will be overcharged, or only ſaturated, I cannot tell.

Case 2. Let the repulſion be inverſely as ſome power of the diſtance between the ſquare and the ſimple power, and let A D be overcharged.

There will be two ſpaces A B and D C, in which the fluid will be preſſed cloſe together, and the quantity of redundant fluid in each of thoſe ſpaces will be more than half the redundant fluid in A D; ſo that the ſpace B C, taken all together, will be undercharged; but I cannot tell in what manner the fluid will be diſpoſed in that ſpace. The demonſtrations of theſe two caſes are exactly ſimilar to thoſe of the two caſes of Prob. 2.

Case 3. If the repulſion is inverſely as the ſimple or ſome lower power of the diſtance, and A D is overcharged, all the fluid will be collected in the ſpaces A B and C D, and B C will be intirely deprived of fluid. If A D contains juſt fluid enough to ſaturate it, and the repulſion is inverſely as the diſtance, the fluid will remain in equilibrio, in whatever manner it is diſpoſed; provided its denſity is everywhere the ſame, at the ſame diſtance from the planes A *a* and D *d*: but if the repulſion is inverſely as ſome leſs power than the ſimple one, the fluid will be in equilibrio, whether it is either ſpread uniformly, or whether it is all collected in that plane which is in the middle between A *a* and D *d*, or whether it is all collected in the ſpaces A B and C D; but not, I believe, if it is diſpoſed in any other manner.

The demonſtration depends upon this circumſtance; namely, that if the repulſion is inverſely as the diſtance, two ſpaces A B and C D, repel a particle, placed either between them, or on the outſide of them, with the ſame force as if all the matter of thoſe ſpaces was collected in the middle plane between them.

It is needleſs mentioning the three caſes in which A D is undercharged, as the reader will eaſily ſupply the place.

Though the four foregoing problems do not immediately tend to explain the phænomena of electricity, I choſe to inſert them; partly becauſe they ſeem worth engaging our attention in themſelves; and partly becauſe they ſerve, in ſome meaſure, to confirm the truth of ſome of the following propoſitions, in which I am obliged to make uſe of a leſs accurate kind of reaſoning.

In the following propoſitions, I ſhall always ſuppoſe the bodies I ſpeak of to conſiſt of ſolid matter, confined to the ſame ſpot, ſo as not to be able to alter its ſhape or ſituation by the attraction or repulſion of other bodies on it: I ſhall alſo ſuppoſe the electric fluid in theſe bodies to be moveable, but unable to eſcape, unleſs when otherwiſe expreſſed. As for the matter in all the reſt of the univerſe, I ſhall ſuppoſe it to be ſaturated with immoveable fluid. I ſhall alſo ſuppoſe the electric attraction and repulſion to be inverſely as any power of the diſtance leſs than the cube, except when otherwiſe expreſſed.

By a canal, I mean a ſlender thread of matter, of ſuch kind that the electric fluid ſhall be able to move readily along it, but ſhall not be able to eſcape from it, except at the ends, where it communicates with other bodies. Thus, when I ſay that two bodies communicate with each other by a canal, I mean that the fluid ſhall be able to paſs readily from one body to the other by that canal.

PROP. IX.

If any body at a diſtance from any over or undercharged body be overcharged, the fluid within it will be lodged in greater quantity near the ſurface of the body than near the center. For, if you ſuppoſe it to be ſpread uniformly all over the body, a particle of fluid in it, near the ſurface, will be repelled towards the ſurface, by a greater quantity of fluid than that by which it is repelled from it; conſequently, the fluid will flow towards the ſurface, and make it denſer there: moreover, the particles of fluid cloſe to the ſurface will be preſſed cloſe together; for otherwiſe, a particle placed ſo near it, that the quantity of redundant fluid between it and the ſurface ſhould be very ſmall, would move towards it; as the ſmall quantity of redundant fluid between it and the ſurface would be unable to balance the repulſion of that on the other ſide.

From the four foregoing problems it ſeems likely, that if the electric attraction or repulſion is inverſely as the ſquare of the diſtance, almoſt all the redundant fluid in the body will be lodged cloſe to the ſurface, and there preſſed cloſe together, and the reſt of the body will be ſaturated. If the repulſion is inverſely as ſome power of the diſtance between the ſquare and the cube, it is likely that all parts of the body will be overcharged: and if it is inverſely as ſome leſs power than the ſquare, it is likely that all parts of the body, except thoſe near the ſurface, will be undercharged.

COROLLARY.

For the ſame reaſon, if the body is undercharged, the deficiency of fluid will be greater near the ſurface than near the center, and the matter near the ſurface will be intirely deprived of fluid. It is likely too, if the repulſion is inverſely as ſome higher power of the diſtance than the ſquare, that all parts of the body will be undercharged: if it is inverſely as the ſquare, that all parts, except near the ſurface, will be ſaturated: and if it is inverſely as ſome leſs power than the ſquare, that all parts, except near the ſurface, will be overcharged.

PROP. X.

Let the bodies A and D (fig. 5.) communicate with each other, by the canal EF; and let one of them, as D, be overcharged; the other body A will be ſo alſo.

For as the fluid in the canal is repelled by the redundant fluid in D, it is plain, that unleſs A was overcharged, ſo as to balance that repulſion, the fluid would run out of D into A.

In like manner, if one is undercharged, the other muſt be ſo too.

PROP.

PROP. XI.

Let the body A (fig. 6.) be either ſaturated or over or undercharged; and let the fluid within it be in equilibrio. Let now the body B, placed near it, be rendered overcharged, the fluid within it being ſuppoſed immoveable, and diſpoſed in ſuch manner, that no part of it ſhall be undercharged; the fluid in A will no longer be in equilibrio, but will be repelled from B: therefore, the fluid will flow from thoſe parts of A which are neareſt to B, to thoſe which are more diſtant from it; and, conſequently, the part adjacent to M N (that part of the ſurface of A which is turned towards B) will be made to contain leſs electric fluid than it did before, and that adjacent to the oppoſite ſurface R S will contain more than before.

It muſt be obſerved, that when a ſufficient quantity of fluid has flowed from M N towards R S, the repulſion which the fluid in the part adjacent to M N exerts on the reſt of the fluid in A, will be ſo much weakened, and the repulſion of that in the part near R S will be ſo much increaſed, as to compenſate the repulſion of B, which will prevent any more fluid flowing from M N to R S.

The reaſon why I ſuppoſe the fluid in B to be immoveable is, that otherwiſe a queſtion might ariſe, whether the attraction or repulſion of the body A might not cauſe ſuch an alteration in the diſpoſition of the fluid in B, as to cauſe ſome parts of it to be undercharged; which might make it doubtful, whether B did on the whole repel the fluid in A. It is evident, however, that this propoſition would hold good, though ſome parts of B were undercharged, provided it did on the whole repel the fluid in A.

COROLLARY.

If B had been made undercharged, inſtead of overcharged, it is plain that ſome fluid would have flowed from the further part R S to the nearer part M N, inſtead of from M N to R S.

PROP. XII.

Let us now ſuppoſe that the body A communicates by the canal E F, with another body D, placed on the contrary ſide of it from B, as in fig. 5; and let theſe two bodies be either ſaturated, or over or undercharged; and let the fluid within them be in equilibrio. Let now the body B be overcharged: it is plain that ſome fluid will be driven from the nearer part M N to the further part R S, as in the former propoſition; and alſo ſome fluid will be driven from R S, through the canal, to the body D; ſo that the quantity of fluid in D will be increaſed thereby, and the quantity in A, taking the whole body together, will be diminiſhed; the quantity in the part near M N will alſo be diminiſhed; but whether the quantity in the part near R S will be diminiſhed or not, does not appear for certain; but I ſhould imagine it would be not much altered.

COROL-

COROLLARY.

In like manner, if B is made undercharged, ſome fluid will flow from D to A, and alſo from that part of A near RS, to the part near MN.

PROP. XIII.

Suppoſe now that the bodies A and D communicate by the bent canal M P N *n p m* (fig. 7.) inſtead of the ſtraight one EF: let the bodies be either ſaturated or over or undercharged as before; and let the fluid be at reſt; then if the body B is made overcharged, ſome fluid will ſtill run out of A into D; provided the repulſion of B on the fluid in the canal is not too great.

The repulſion of B on the fluid in the canal, will at firſt drive ſome fluid out of the leg MP*pm* into A, and out of NP*pn* into D, till the quantity of fluid in that part of the canal which is neareſt to B is ſo much diminiſhed, and its repulſion on the reſt of the fluid in the canal is ſo much diminiſhed alſo as to compenſate the repulſion of B: but as the leg NP*pn* is longer than the other, the repulſion of B on the fluid in it will be greater; conſequently ſome fluid will run out of A into D, on the ſame principle that water is drawn out of a veſſel through a ſyphon: but if the repulſion of B on the fluid in the canal is ſo great, as to drive all the fluid out of the ſpace GPH*p*G, ſo that the fluid in the leg MG*pm* does not

join

join to that in N H *pn*; then it is plain that no fluid can run out of A into D; any more than water will run out of a veſſel through a ſyphon, if the height of the bend of the ſyphon above the water in the veſſel, is greater than that to which water will riſe in vacuo.

COROLLARY.

If B is made undercharged, ſome fluid will run out of D into A; and that though the attraction of B on the fluid in the canal is ever ſo great.

PROP. XIV.

Let ABC (fig. 8.) be a body overcharged with immoveable fluid, uniformly ſpread; let the bodies near ABC on the outſide be ſaturated with immoveable fluid; and let D be a body incloſed within ABC, and communicating by the canal DG with other diſtant bodies ſaturated with fluid; and let the fluid in D and the canal and thoſe bodies be moveable; then will the body D be rendered undercharged.

For let us firſt ſuppoſe that D and the canal are ſaturated, and that D is nearer to B than to the oppoſite part of the body, C; then will all the fluid in the canal be repelled from C by the redundant fluid in ABC; but if D is nearer to C than to B, take the point F, ſuch that a particle placed there would be repelled from C with as much force as one at D is repelled towards C; the fluid in DF, taking the whole together, will be repelled with as much force one way as the other; and the fluid in FG is all of it repelled from C: therefore in both caſes the fluid in the canal, taking the whole together, is repelled from C; conſequently ſome fluid will run out of D and the canal, till the attraction of the unſaturated matter therein is ſufficient to balance the repulſion of the redundant fluid in ABC.

PROP. XV.

If we now ſuppoſe that the fluid on the outſide of ABC is moveable; the matter adjacent to ABC on the outſide, will become undercharged. I ſee no reaſon however to think that that will prevent the body D from being undercharged; but I cannot ſay exactly what effect it will have, except when ABC is ſpherical and the repulſion is inverſely as the ſquare of the diſtance; in this caſe it appears by Prob. I. that the fluid in the part DB of the canal will be repelled from C, with juſt as much force as in the laſt propoſition; but the fluid in the part BG will not be repelled at all: conſequently D will be undercharged, but not ſo much as in the laſt propoſition.

COROLLARY.

If ABC is now ſuppoſed to be undercharged, it is certain that D will be overcharged, provided the matter near ABC on the outſide is ſaturated with immoveable

moveable fluid; and there is great reaſon to think that it will be ſo, though the fluid in that matter is moveable.

PROP. XVI.

Let AEFB (fig. 9.) be a long cylindric body, and D an undercharged body; and let the quantity of fluid in AEFB be ſuch, that the part near EF ſhall be ſaturated. It appears from what has been ſaid before, that the part near AB will be overcharged; and moreover there will be a certain ſpace, as A*ab*B, adjoining to the plane AB, in which the fluid will be preſſed cloſe together; and the fluid in that ſpace will preſs againſt the plane A B, and will endeavour to eſcape from it; and by Prop. II. the two bodies will attract each other: now I ſay that the force with which the fluid preſſes againſt the plane AB, is very nearly the ſame with which the two bodies attract each other in the direction EA; provided that no part of AEFB is undercharged.

Suppoſe ſo much of the fluid in each part of the cylinder as is ſufficient to ſaturate the matter in that part, to become ſolid; the remainder, or the redundant fluid remaining fluid as before. In this caſe the preſſure againſt the plane AB muſt be exactly equal to that with which the two bodies attract each other, in the direction EA: for the force with which D attracts that part of the fluid which we ſuppoſed to become ſolid, is exactly equal to that, with which it repels the matter in the cylinder; and the redundant fluid in E *ab* F is at liberty to move, if it had any tendency to do ſo, without moving the cylinder; ſo that the only thing which has any tendency to impel the cylinder in the direction E A is the preſſure of the redundant fluid in A *ab* B againſt AB; and as the part near E F is ſaturated, there is no redundant fluid to preſs againſt the plane E F, and thereby to counteract the preſſure againſt A B. Suppoſe now all the electric fluid in the cylinder to become fluid; the force with which the two bodies attract each other will remain exactly the ſame; and the only alteration in the preſſure againſt A B, will be, that that part of the fluid in A *ab* B, which we at firſt ſuppoſed ſolid and unable to preſs againſt the plane, will now be at liberty to preſs againſt it; but as the denſity of the fluid when its particles are preſſed cloſe together may be ſuppoſed many times greater than when it is no denſer than ſufficient to ſaturate the matter in the cylinder, and conſequently the quantity of redundant fluid in A *ab* B many times greater than that which is required to ſaturate the matter therein, it follows that the preſſure againſt A B will be very little more than on the firſt ſuppoſition.

N. B. If any part of the cylinder is undercharged, the preſſure againſt A B is greater than the force with which the bodies attract. If the electric repulſion is inverſely as the ſquare or ſome higher power of the diſtance, it ſeems very unlikely that any part of the cylinder ſhould be undercharged; but if the repulſion is inverſely as ſome lower power than the ſquare, it is not improbable but ſome part of the cylinder may be undercharged.

LEMMA VII.

Let AB (fig. 10.) repreſent an infinitely thin flat circular plate, ſeen edgeways, ſo as to appear to the eye as a ſtraight line; let C be the center of the circle; and let DC paſſing through C, be perpendicular to the plane of the plate; and let the plate be of uniform thickneſs, and conſiſt of uniform matter, whoſe particles repel with a force inverſely as the *n* power of the diſtance; *n* being greater than one, and leſs than three: the repulſion of the plate on a particle at D is proportional to $\frac{DC}{DC^{n-1}} - \frac{DC}{DA^{n-1}}$; provided the thickneſs of the plate and ſize of the particle D is given.

For if CA is ſuppoſed to flow, the correſponding fluxion of the quantity of matter in the plate, is proportional to $CA \times C\dot{A}$; and the correſponding fluxion of the repulſion of the plate on the particle D, in the direction DC, is proportional to $\frac{CA \times C\dot{A}}{DA^{n}} \times \frac{DC}{DA}, = \frac{D\dot{A} \times DC}{DA^{n}}$; for $D\dot{A}$ is to $C\dot{A} :: CA : DA$; the variable part of the fluent of which is $\frac{-DC}{n-1 \times DA^{n-1}}$: whence the repulſion of the plate on the particle D is proportional to $\frac{DC}{n-1 \times DC^{n-1}} - \frac{DC}{n-1 \times DA^{n-1}}$, or to $\frac{DC}{DC^{n-1}} - \frac{DC}{DA^{n-1}}$.

COROLLARY.

If DC^{n-1} is very ſmall in reſpect of CA^{n-1}, the particle D is repelled with very nearly the ſame force as if the diameter of the plate was infinite.

LEMMA VIII.

Let L and l repreſent the two legs of a right angled triangle, and h the hypothenuſe; if the ſhorter leg l is ſo much leſs than the other, that l^{n-1} is very ſmall in reſpect of L^{n-1}, $h^{3-n} - L^{3-n}$ will be very ſmall in reſpect of l^{3-n}.

For $h^{3-n} = \overline{L^2+l^2}|^{\frac{3-n}{2}}$, $= L^{3-n} \times \overline{1+\frac{l^2}{L^2}}|^{\frac{3-n}{2}}$, $=$ $L^{3-n} \times 1 + \frac{\overline{3-n} \times l^2}{2L^2} - \frac{\overline{3-n} \times \overline{n-1} \times l^4}{8L^4}$, &c. therefore $h^{3-n} - L^{3-n} = \frac{\overline{3-n} \times l^2}{2L^{n-1}} - \frac{\overline{3-n} \times \overline{n-1} \times l^4}{8L^{n+1}}$, &c. $=$ $\frac{l^{3-n} \times \overline{3-n} \times l^{n-1}}{2L^{n-1}} - \frac{l^{3-n} \times \overline{3-n} \times \overline{n-1} \times l^{n+1}}{8L^{n+1}}$, &c. which is very ſmall in reſpect of l^{3-n}; as l^{n-1} is by the ſuppoſition very ſmall in reſpect of L^{n-1}.

LEMMA IX.

Let DC now repreſent the axis of a cylindric or priſmatic column of uniform matter; and let the diameter of the column be ſo ſmall, that the repulſion of the plate AB on it ſhall not be ſenſibly different from what it would be, if all the matter in it was collected in the axis: the force with which the plate repells the column, is proportional to $DC^{3-n}+AC^{3-n}-DA^{3-n}$; ſuppoſing the thickneſs of the plate and baſe of the column to be given.

For, if DC is ſuppoſed to flow, the correſponding fluxion of the repulſion is proportional to $\frac{D\dot{C}}{DC^{n-2}} - \frac{DC\times D\dot{C}}{DA^{n-1}} = \frac{D\dot{C}}{DC^{n-2}} - \frac{D\dot{A}}{DA^{n-2}}$; the fluent of which, $\frac{AC^{3-n}+DC^{3-n}-DA^{3-n}}{3-n}$, vaniſhes when DC vaniſhes.

COROLL. I.

If the length of the column is ſo great that AC^{n-1} is very ſmall in reſpect of DC^{n-1}, the repulſion of the plate on it is very nearly the ſame as if the column was infinitely continued.

For by Lemma 8, $AC^{3-n}+DC^{3-n}-DA^{3-n}$ differs very little in this caſe from AC^{3-n}; and if DC is infinite, it is exactly equal to it.

COROLL. II

If AC^{n-1} is very ſmall in reſpect of DC^{n-1}; and the point E be taken in DC ſuch that EC^{n-1}, ſhall be very ſmall in reſpect of AC^{n-1}, the repulſion of the plate on the ſmall part of the column EC, is to its repulſion on the whole column DC, very nearly as EC^{3-n} to AC^{3-n}.

LEMMA X.

If we now ſuppoſe all the matter of the plate to be collected in the circumference of the circle, ſo as to form an infinitely ſlender uniform ring, its repulſion on the column DC will be leſs than when the matter is ſpread uniformly all over the plate, in the ratio of $\frac{3-n\times AC^{2}}{2}\times\overline{\frac{1}{AC^{n-1}}-\frac{1}{DA^{n-1}}}$ to $DC^{3-n}+AC^{3-n}-DA^{3-n}$.

For it was before ſaid, that if the matter of the plate be ſpread uniformly, its repulſion on the column will be proportional to $DC^{3-n}+AC^{3-n}-DA^{3-n}$, or may be expreſſed thereby; let now AC, the ſemidiameter of the plate, be increaſed by the infinitely ſmall quantity $A\dot{c}$; the quantity of matter in the plate will be increaſed by a quantity, which is to the whole, as $2A\dot{c}$ to AC; and the repulſion of the plate on the column, will be increaſed by $3-n\times A\dot{c}\times AC^{2-n}-A\dot{c}\times\frac{AC}{DA}\times 3-n\times DA^{2-n}, =3-n\times A\dot{c}\times AC\times\overline{\frac{1}{AC^{n-1}}-\frac{1}{DA^{n-1}}}$: therefore if a quantity of matter, which is to the whole quantity in the plate, as $2A\dot{c}$ to AC be collected in the circumference, its repulſion on the column DC, will be to that of the whole plate, as $3-n\times A\dot{c}\times AC\times\overline{\frac{1}{AC^{n-1}}-\frac{1}{DA^{n-1}}}$, to $DC^{3-n}+AC^{3-n}-DA^{3-n}$; and conſequently the repulſion of the plate when all the matter is collected in its circumference, is to its repulſion when the matter is ſpread uniformly, as $\frac{3-n\times AC^2}{2}\times\overline{\frac{1}{AC^{n-1}}-\frac{1}{DA^{n-1}}}$, to $DC^{3-n}+AC^{3-n}-DA^{3-n}$.

COROLL. I.

If the length of the column is ſo great, that AC^{n-1} is very ſmall in reſpect of DC^{n-1}, the repulſion of the plate, when all the matter is collected in the circumference, is to its repulſion when the matter is ſpread uniformly, very nearly as $\frac{3-n\times AC^{3-n}}{2}$ to AC^{3-n}, or as $3-n$ to 2.

COROLL. II.

If EC^{n-1} is very ſmall in reſpect of AC^{n-1}, the repulſion of the plate on the ſhort column EC, when all the matter in the plate is collected in its circumference, is to its repulſion when the matter is ſpread uniformly, very nearly as $\frac{3-n\times n-1\times EC^2}{4AC^{n-1}}$ to EC^{3-n}, or as $3-n\times n-1\times EC^{n-1}$ to $4\,AC^{n-1}$; and is therefore very ſmall in compariſon of what it is when the matter is ſpread uniformly.

For by the ſame kind of proceſs as was uſed in Lemma 8, it appears, that if EC^2 is very ſmall in reſpect of AC^2, $AC^2\times\overline{\frac{1}{AC^{n-1}}-\frac{1}{EA^{n-1}}}$ differs very little from $\frac{n-1\times EC^2}{2EA^{n-1}}$, or from $\frac{n-1\times EC^2}{2AC^{n-1}}$; and if EC^{n-1} is very ſmall in reſpect of AC^{n-1}, EC^2 is *a fortiori* very ſmall in reſpect of AC^2.

COROLL. III.

Suppoſe now that the matter of the plate is denſer near the circumference than near the middle, and that the denſity at and near the middle is to the mean denſity, or the denſity which it would everywhere be of if the matter was ſpread uniformly, as δ to one; the repulſion of the plate on EC will be leſs than if the matter was ſpread uniformly, in a ratio approaching much nearer to that of δ to one, than to that of equality.

COROLL. IV.

Let every thing be as in the laſt corollary, and let π be taken to one, as the force with which the plate actually repels the column DC (DC^{n-1} being very great in reſpect of AC^{n-1}) is to the force with which it would repel it, if the matter was ſpread uniformly; the repulſion of the plate on EC will be to its repulſion on DC, in a ratio between that of $EC^{3-n}\times\delta$ to $AC^{3-n}\times\pi$, and that of EC^{3-n} to $AC^{3-n}\times\pi$, but will approach much nearer to the former ratio than to the latter.

LEMMA XI.

In the line DC produced, take CF equal to CA: if all the matter of the plate AB is collected in the circumference, its repulſion on the column CD, infinitely continued, is equal to the repulſion of the ſame quantity of matter collected in the point F, on the ſame column.

For the repulſion of the plate on the column in the direction CD, is the ſame, whether the matter of it be collected in the whole circumference, or in the point A. Suppoſe it therefore to be collected in A; and let an equal quantity of matter be collected in F; take FG conſtantly equal to AD; and let AD and FG flow: the fluxion of CD is to the fluxion of FG, as AD to CD; and the repulſion of A on the point D, in the direction CD, is to the repulſion of F on G, as CD to AD; and therefore the fluxion of the repulſion of A on the column CD, in the direction CD, is equal to the fluxion of the repulſion of F on CG; and when AD equals AC, the repulſion of both A and F on their reſpective columns vaniſhes; and therefore the repulſion of A on the whole column CD equals that of F on CG; and when CD and CG are both infinitely extended, they may be looked upon as the ſame column.

PROP. XVII.

Let two ſimilar bodies, of different ſizes, and conſiſting of different ſorts of matter, be both over-

charged,

Fig. 11.

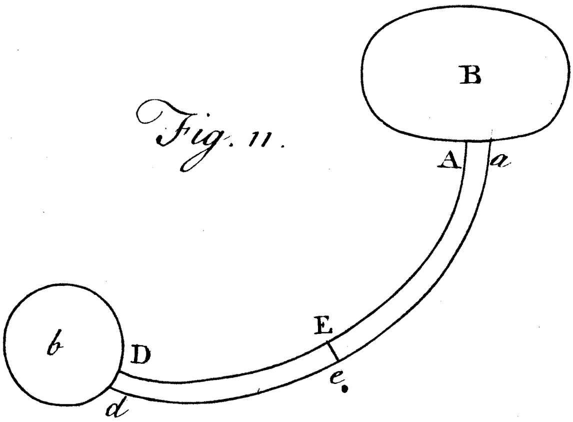


Fig. 12.

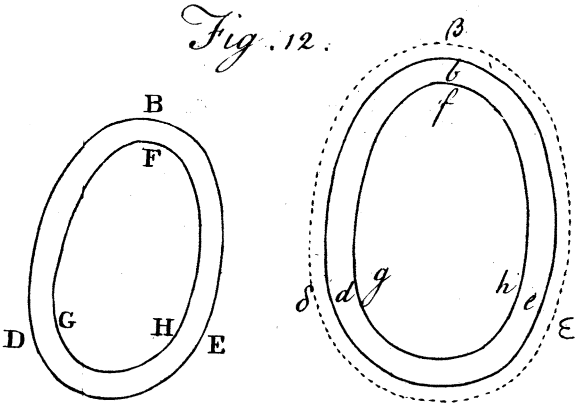


Fig. 13.

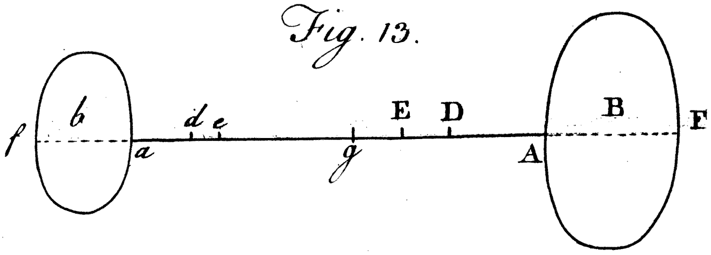


Fig. 15

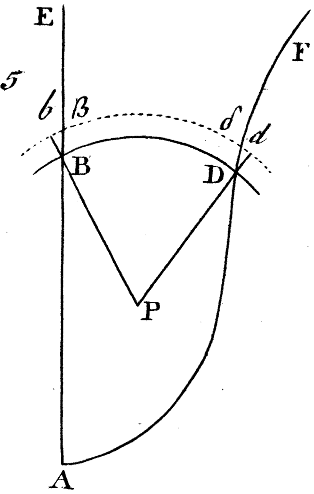


Fig. 14.

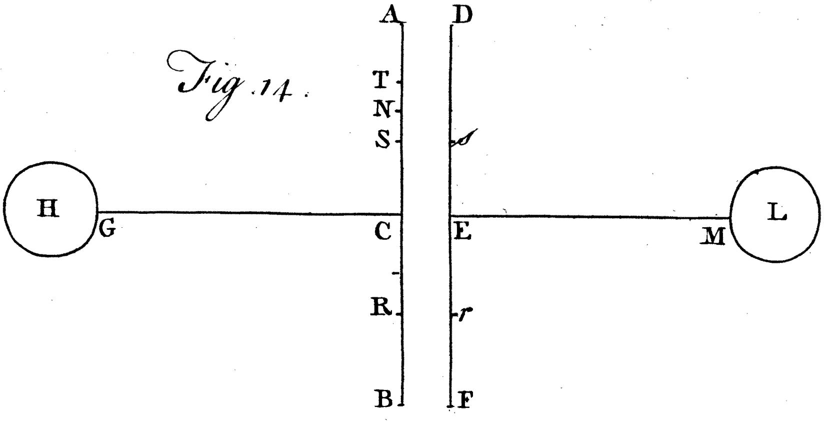


Fig. 16.

:16.

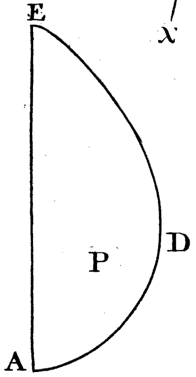
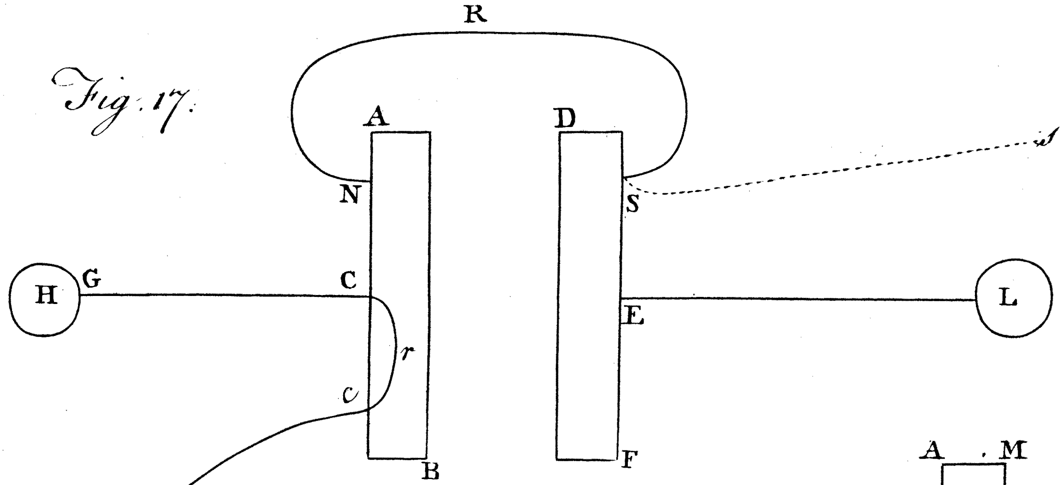
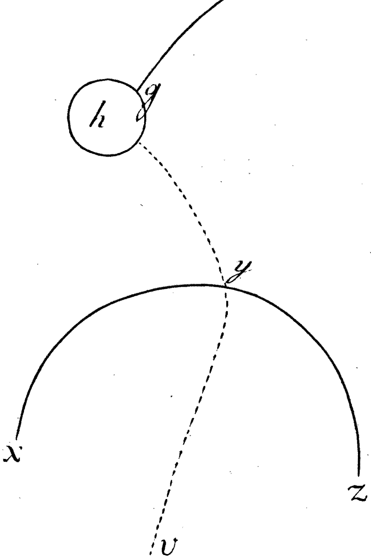


Fig:17.





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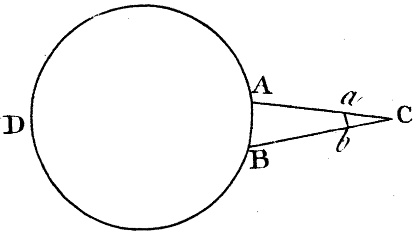


Fig.19.

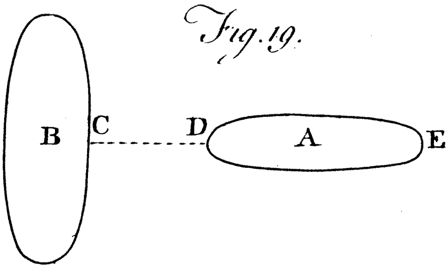
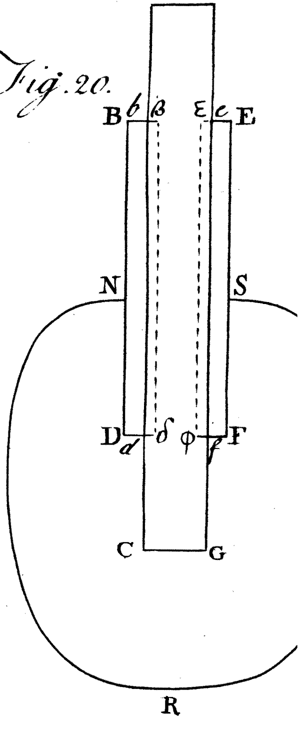
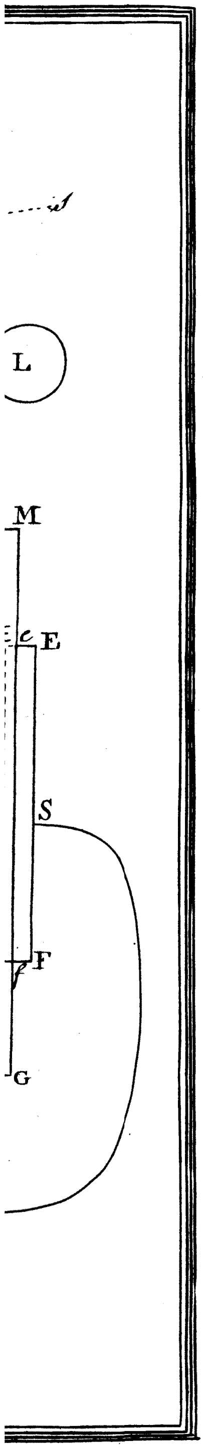


Fig.20.





Basire Sc.

charged, or both undercharged, but in different degrees; and let the redundance or deficience of fluid in each be very ſmall in reſpect of the whole quantity of fluid in them: it is impoſſible for the fluid to be diſpoſed accurately in a ſimilar manner in both of them *; as it has been ſhewn that there will be a ſpace, cloſe to the ſurface, which will either be as full of fluid as it can hold, or will be intirely deprived of fluid; but it will be diſpoſed as nearly in a ſimilar manner in both, as is poſſible. To explain this, let B D E and *b d e* (fig. 12) be the two ſimilar bodies; and let the ſpace comprehended between the ſurfaces B D E and F G H (or the ſpace B F as I ſhall call it for ſhortneſs) be that part of BDE, which is either as full of fluid as it can hold, or intirely deprived of it: draw the ſurface *f g h*, ſuch that the ſpace *b f*, ſhall be to the ſpace B F, as the quantity of redundant or deficient fluid in *b d e*, to that in B D E, and that the thickneſs of the ſpace *b f* ſhall every where bear the ſame proportion to the correſponding thickneſs of B F: then will the ſpace *b f* be either as full of fluid as it can hold, or intirely deprived of it; and the fluid within the ſpace fgh will be diſpoſed very nearly ſimilarly to that in the ſpace FGH.

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* By the fluid being diſpoſed in a ſimilar manner in both bodies, I mean that the quantity of redundant or deficient fluid in any ſmall part of one body, is to that in the correſponding ſmall part of the other, as the whole quantity of redundant or deficient fluid in one body, to that in the other. By the quantity of deficient fluid in a body, I mean the quantity of fluid wanting to ſaturate it. Notwithſtanding the impropriety of this expreſſion, I muſt beg leave to make uſe of it, as it will frequently ſave a great deal of circumlocution.

For it is plain, that if the fluid could be diſpoſed accurately in a ſimilar manner in both bodies, the fluid would be in equilibrio in one body, if it was in the other: therefore draw the ſurface $\beta\delta\epsilon$, ſuch that the thickneſs of the ſpace βf ſhall be every where to the correſponding thickneſs of BF, as the diameter of bde to the diameter of BDE; and let the redundant fluid or matter in bf be ſpread uniformly over the ſpace βf; then if the fluid in the ſpace fgh is diſpoſed exactly ſimilarly to that in FGH, it will be in equilibrio; as the fluid will then be diſpoſed exactly ſimilarly in the ſpaces $\beta\delta\epsilon$ and BDE: but as by the ſuppoſition, the thickneſs of the ſpace βf is very ſmall in reſpect of the diameter of bde, the fluid or matter in the ſpace bf will exert very nearly the ſame force on the reſt of the fluid, whether it is ſpread over the ſpace βf, or whether it is collected in bf.

PROP. XVIII.

Let two bodies, B and b, be connected to each other by a canal of any kind, and be either over or undercharged: it is plain that the quantity of redundant or deficient fluid in B, would bear exactly the ſame proportion to that in b, whatever ſort of matter B conſiſted of, if it was poſſible for the redundant or deficient fluid in any body, to be diſpoſed accurately in the ſame manner, whatever ſort of matter it conſiſted of. For ſuppoſe B to conſiſt of any ſort of matter; and let the fluid in the canal and two bodies be in equilibrio: let now B be made to conſiſt of ſome other ſort of matter, which requires a different quantity of fluid to ſaturate it; but let the quantity and diſpoſition of the redundant or deficient fluid in it remain the ſame as before: it is plain that the fluid will ſtill be in equilibrio; as the attraction or repulſion of any body depends only on the quantity and diſpoſition of the redundant and deficient fluid in it. Therefore, by the preceeding propoſition, the quantity of redundant or deficient fluid in B, will actually bear very nearly the ſame proportion to that in *b*, whatever ſort of matter B conſiſts of; provided the quantity of redundant or deficient fluid in it is very ſmall in reſpect of the whole.

PROP. XIX.

Let two bodies B and *b* (fig. 11.) be connected together by a very ſlender canal AD*da*, either ſtraight or crooked: let the canal be everywhere of the ſame breadth and thickneſs; ſo that all ſections of this canal made by planes perpendicular to the direction of the canal in that part, ſhall be equal and ſimilar: let the canal be compoſed of uniform matter; and let the electric fluid therein be ſuppoſed incompreſſible, and of ſuch denſity as exactly to ſaturate the matter therein; and let it, nevertheleſs, be able to move readily along the canal; and let each particle of fluid in the canal be attracted and repelled by the matter and fluid in the canal and in the bodies B and *b*, juſt in the ſame manner that it would be if it was not incompreſſible *; and let the bodies B and *b* be either over or undercharged. I ſay that the force with which the whole quantity of fluid in the canal is impelled from A towards D, in the direction of the axis of the canal, by the united attractions and repulſions of the two bodies, muſt be nothing; as otherwiſe the fluid in the canal could not be at reſt: obſerving that by the force with which the whole quantity of fluid is impelled in the direction of the axis of the canal, I mean the ſum of the forces, with which the fluid in each part of the canal is impelled in the direction of the axis of the canal in that place, from A towards D; and obſerving alſo, that an impulſe in the contrary direction from D towards A muſt be looked upon as negative.

For as the canal is exactly ſaturated with fluid, the fluid therein is attracted or repelled only by the redundant matter or fluid in the two bodies. Suppoſe now that the fluid in any ſection of the canal, as E *e*, is impelled with any given force in the direction of the canal at that place, the ſection D *d* would, in conſequence thereof, be impelled with exactly the ſame force in the direction or the canal at D, if the fluid between E *e* and D *d* was not at all attracted or repelled by the two bodies; and, conſequently, the ſection D *d* is impelled in the direction of the canal, with the ſum of the forces, with which the fluid in each part of the canal is impelled, by the attraction or repulſion of the two bodies in the direction of the axis in that part; and conſequently, unleſs this ſum was nothing, the fluid in D *d* could not be at reſt.

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* This ſuppoſition of the fluid in the canal being incompreſſible, is not mentioned as a thing which can ever take place in nature, but is merely imaginary; the reaſon for making of which will be given hereafter.

COROLLARY.

Therefore, the force with which the fluid in the canal is impelled one way in the direction of the axis, by the body B, muſt be equal to that with which it impelled by *b* in the contrary direction.

PROP. XX.

Let two ſimilar bodies B and *b* (fig. 13.) be connected by the very ſlender cylindric or priſmatic canal A *a*, filled with incompreſſible fluid, in the ſame manner as deſcribed in the preceding propoſition: let the bodies be overcharged; but let the quantity of redundant fluid in each bear ſo ſmall a proportion to the whole, that the fluid may be conſidered as diſpoſed in a ſimilar manner in both; let the bodies alſo be ſimilarly ſituated in reſpect of the canal A *a*; and let them be placed at an infinite diſtance from each other, or at ſo great an one, that the repulſion of either body on the fluid in the canal ſhall not be ſenſibly leſs than if they were at an infinite diſtance: then, if the electric attraction and repulſion is inverſely as the n power of the diſtance, n being greater than one, and leſs than three, the quantity of redundant fluid in the two bodies will be to each other, as the $n-1$ power of their correſponding diameters AF and *af*.

For if the quantity of redundant fluid in the two bodies is in this proportion, the repulſion of one body on the fluid in the canal, will be equal to that of the other body on it in the contrary direction; and, conſequently, the fluid will have no tendency to flow from one body to the other, as may thus be proved. Take the points D and E very near to each other; and take *da* to DA, and *ea* to EA, as *af* to AF; the repulſion of the body B on a particle at D, will be to the repulſion of *b* on a particle at *d*, as $\frac{1}{AF}$ to $\frac{1}{af}$; for, as the fluid is diſpoſed ſimilarly in both bodies, the quantity of fluid in any ſmall part of B, is to the quantity in the correſponding part of *b*, as AF^{n-1} to af^{n-1}; and, conſequently, the repulſion of that ſmall part of B, on D, is to the repulſion of the correſponding part of *b*, on *d*, as $\frac{AF^{n-1}}{AF^{n}}$, or $\frac{1}{AF}$, to $\frac{1}{af}$. But the quantity of fluid in the ſmall part DE of the canal, is to that in *de*, as DE to *de*, or as AF to *af*; therefore the repulſion of B on the fluid in D E, is equal to that of *b* on the fluid in *de*: therefore, taking *ag* to A *a*, as *af* to A F, the repulſion of *b* on the fluid in *ag*, is equal to that of B on the fluid in A *a*; but the repulſion of *b* on *ag* may be conſidered as the ſame as its repulſion on A *a*; for, by the ſuppoſition, the repulſion of B on A *a* may be conſidered as the ſame as if it was continued infinitely; and therefore, the repulſion of *b* on *ag* may be conſidered as the ſame as if it was continued infinitely.

N. B. If *n* was not greater than one, it would be impoſſible for the length of A *a* to be ſo great, that the repulſion of B on it might be conſidered as the ſame as if it was continued infinitely; which was my reaſon for requiring *n* to be greater than one.

COROLLARY.

By juſt the ſame method of reaſoning it appears, that if the bodies are undercharged, the quantity of deficient fluid in *b* will be to that in B, as af^{n-1} to AF^{n-1}.

PROP. XXI.

Let a thin flat plate be connected to any other body, as in the preceding propoſition, by a canal of incompreſſible fluid, perpendicular to the plane of the plate; and let that body be overcharged, the quantity of redundant fluid in the plate will bear very nearly the ſame proportion to that in the other body, whatever the thickneſs of the plate may be, provided its thickneſs is very ſmall in proportion to its breadth, or ſmalleſt diameter.

For there can be no doubt, but what, under that reſtriction, the fluid will be diſpoſed very nearly in the ſame manner in the plate, whatever its thickneſs may be; and therefore its repulſion on the fluid in the canal will be very nearly the ſame, whatever its thickneſs may be.

PROP. XXII.

Let AB and DF (fig. 14.) repreſent two equal and parallel circular plates, whoſe centers are C and E; let the plates be placed ſo, that a right line joining their centers ſhall be perpendicular to the plates; let the thickneſs of the plates be very ſmall, in reſpect of their diſtance CE; let the plate AB communicate with the body H, and the plate DF with the body L, by the canals CG and EM of incompreſſible fluid, ſuch as are deſcribed in Prop. XIX; let theſe canals meet their reſpective plates in their centers C and E, and be perpendicular to the plane of the plates; and let their length be ſo great, that the repulſion of the plates on the fluid in them may be conſidered as the ſame, as if they were continued infinitely; let the body H be overcharged, and let L be ſaturated. It is plain, from Prop. XII. that DF will be undercharged, and AB will be more overcharged

charged than it would otherwiſe be. Suppoſe, now, that the redundant fluid in AB is diſpoſed in the ſame manner as the deficient fluid is in DF; let P be to one as the force with which the plate AB would repel the fluid in CE, if the canal ME was continued to C, is to the force with which it would repel the fluid in CM; and let the force with which AB repels the fluid in CG, be to the force with which it would repel it, if the redundant fluid in it was ſpread uniformly, as π to 1; and let the force with which the body H repels the fluid in CG, be the ſame with which a quantity of redundant fluid, which we will call B, ſpread uniformly over AB, would repel it in the contrary direction. Then will the redundant fluid in AB be equal to $\frac{B}{2P\pi - P^2\pi}$, and therefore, if P is very ſmall, will be very nearly equal to $\frac{B}{2P\pi}$; and the deficient fluid in DF will be to the redundant fluid in AB, as $1 - P$ to one, and therefore, if P is very ſmall, will be very nearly equal to the redundant fluid in AB.

For it is plain, that the force with which AB repels the fluid in EM, muſt be equal to that with which DF attracts it; for otherwiſe, ſome fluid would run out of DF into L, or out of L into DF: for the ſame reaſon, the exceſs of the repulſion of AB on the fluid in CG, above the attraction of FD thereon, muſt be equal to the force with which a quantity of redundant fluid equal to B, ſpread uniformly over A B, would repel it, or it muſt be equal to that with which a quantity equal to $\frac{B}{\pi}$, ſpread in the manner in which the redundant fluid is actually ſpread in A B, would repel it. By the ſuppoſition, the force with which A B repels the fluid in E M, is to the force with which it would repel the fluid in C M, ſuppoſing E M to be continued to C, as $1 - P$ to one; but the force with which any quantity of fluid in A B would repel the fluid in C M, is the ſame with which an equal quantity ſimilarly diſpoſed in D F, would repel the fluid in E M; therefore, the force with which the redundant fluid in A B repels the fluid in E M, is to that with which an equal quantity ſimilarly diſpoſed in D F, would repel it, as $1 - P$ to one: therefore, if the redundant fluid in A B be called A, the deficient fluid in D F muſt be $A \times 1 - P$: for the ſame reaſon, the force with which D F attracts the fluid in C G, is to that with which A B repels it, as $A \times 1 - P \times 1 - P$, or $A \times \overline{1 - P}^2$, to A; therefore, the exceſs of the force with which A B repels C G above that with which D F attracts it, is equal to that with which a quantity of redundant fluid equal to $A - \overline{A \times \overline{1 - P}^2}$, or $\overline{A \times 2P - P^2}$, ſpread over A B, in the manner in which the redundant fluid therein is actually ſpread, would repel it: therefore, $\overline{A \times 2P - P^2}$ muſt be equal to $\frac{B}{\pi}$, or A muſt be equal to $\frac{B}{2P\pi - P^2\pi}$.

COROL. I.

If the denſity of the redundant fluid near the middle of the plate A B, is leſs than the mean denſity, or the denſity which it would everywhere be of, if it was ſpread uniformly, in the ratio of δ to one; and if the diſtance of the two plates is ſo ſmall, that EC^{n-1} is very ſmall in reſpect of AC^{n-1}, and that EC^{3-n} is very ſmall in reſpect of AC^{3-n}, the quantity of redundant fluid in AB will be greater than $\frac{B}{2} \times \overline{\frac{AC}{EC}}\Big|^{3-n}$, and leſs than $\frac{B}{2\delta} \times \overline{\frac{AC}{EC}}\Big|^{3-n}$, but will approach much nearer to the latter value than the former. For, in this caſe, $P\pi$ is, by Lemma X. Corol. IV. leſs than $\overline{\frac{EC}{AC}}\Big|^{3-n}$, and greater than $\overline{\frac{EC}{AC}}\Big|^{3-n} \times \delta$, but approaches much nearer to the latter value than the former; and if EC^{3-n} is very ſmall in reſpect of AC^{3-n}, P is very ſmall.

REMARKS.

If DF was not undercharged, it is certain that AB would be conſiderably more overcharged near the circumference of the circle than near the center; for if the fluid was ſpread uniformly, a particle placed anywhere at a diſtance from the center, as at N, would be repelled with conſiderably more force towards the circumference than it would towards the center. If the plates are very near together, and, conſequently, DF nearly as much undercharged as AB is overcharged, AB will ſtill be more overcharged near the circumference than near the center, but the difference will not be near ſo great as in the former caſe: for, let NR be many times greater than CE, and NS leſs than CE; and take E*r* and E*s* equal to CR and CS, there can be no doubt, I think, but that the deficient fluid in DF will be lodged nearly in the ſame manner as the redundant fluid in AB; and therefore, the repulſion of the redundant fluid at R, on a particle at N, will be very nearly balanced by the attraction of the redundant matter at *r*, for R is not much nearer to N than *r* is; but the repulſion of S will not be near balanced by that of *s*; for the diſtance of S from N is much leſs than that of *s*. Let now a ſmall circle, whoſe diameter is ST, be drawn round the center N, on the plane of the plate; as the denſity of the fluid is greater at T than at S, the repulſion of the redundant fluid within the ſmall circle tends to impel the point N towards C; but as there is a much greater quantity of fluid between N and B, than between N and A, the repulſion of the fluid without the ſmall circle tends to balance that; but the effect of the fluid within the ſmall circle is not much leſs than it would be, if DF was not undercharged; whereas much the greater part of the effect of that part of the plate on the outſide of the circle, is taken off by the effect of the correſponding part of DF: conſequently, the difference of denſity between T and S will not be near ſo great, as if DF was not undercharged. Hence I ſhould imagine, that if the two plates are very near together, the denſity of the redundant fluid near the center will not be much leſs than the mean denſity, or δ will not be much leſs than one; moreover, the leſs the diſtance of the plates, the nearer will δ approach to one.

Corol. II.

Let now the body H conſiſt of a circular plate, of the ſame ſize as A B, placed ſo, that the canal C G ſhall paſs through its center, and be perpendicular to its plane; by the ſuppoſition, the force with which H repels the fluid in the canal C G, is the ſame with which a quantity of fluid, equal to B, ſpread uniformly over A B, would repel it in the contrary direction: therefore, if the fluid in the plate H was ſpread uniformly, the quantity of redundant fluid therein would be B, and if it was all collected in the circumference, would be $\frac{2B}{3-n}$; and therefore the real quantity will be greater than B, and leſs than $\frac{2B}{3-n}$.

Corol. III.

Therefore, if we ſuppoſe δ to be equal to one, the quantity of redundant fluid in A B will exceed that in the plate H, in a greater ratio than that of $\left.\overline{\frac{AC}{CE}}\right|^{3-n} \times \frac{3-n}{4}$ to one, and leſs than that of $\left.\overline{\frac{AC}{CE}}\right|^{3-n} \times \frac{1}{2}$ to one; and from the preceding remarks it appears, that the real quantity of redundant fluid in A B can hardly be much greater than it would if δ was equa to one.

Corol. IV.

Hence, if the electric attraction and repulſion is inverſely as the ſquare of the diſtance, the redundant fluid in A B, ſuppoſing δ to be equal to one, will exceed that in the plate H, in a greater ratio than that of A C to 4 C E, and leſs than that of A C to 2 C E.

Corol. V.

Let now the body H conſiſt of a globe, whoſe diameter equals A B; the globe being ſituated in ſuch a manner, that the canal C G, if continued, would paſs through its center; and let the electric attraction and repulſion be inverſely as the ſquare of the diſtance, the quantity of redundant fluid in the globe will be 2 B: for the fluid will be ſpread uniformly over the ſurface of the globe, and its repulſion on the canal will be the ſame as if it was all collected in the center of the ſphere, and will therefore be the ſame with which an equal quantity, diſpoſed in the circumference of A B, would repel it in the contrary direction, or with which half that quantity, or B, would repel it, if ſpread uniformly over the plate.

COROL. VI.

Therefore, if δ was equal to one, the redundant fluid in AB would exceed that in the globe, in the ratio of AC to 4CE; and therefore, it will in reality exceed that in the globe, in a rather greater ratio than that of AC to 4CE; but if the plates are very near together, it will approach very near thereto, and the nearer the plates are, the nearer it will approach thereto.

COROL. VII.

Whether the electric repulſion is inverſely as the ſquare of the diſtance or not, if the body H is as much undercharged, as it was before overcharged, AB will be as much undercharged as it was before overcharged, and DF as much overcharged as it was before undercharged.

COROL. VIII.

If the ſize and diſtance of the plates be altered, the quantity of redundant or deficient fluid in the body H remaining the ſame, it appears, by comparing this propoſition with the 20th and 21ſt propoſitions, that the quantity of redundant and deficient fluid in AB will be as $AC^{n-1} \times \overline{\left.\frac{AC}{EC}\right|}^{3-n}$, or as $\frac{AC^{2}}{EC^{3-n}}$, ſuppoſing the value of δ to remain the ſame.

PROP.

PROP. XXIII.

Let AE (fig. 15.) be a cylindric canal, infinitely continued beyond E; and let AF be a bent canal, meeting the other at A, and infinitely continued beyond F: ſet the ſection of this canal, in all parts of it, be equal to that of the cylindric canal, and let both canals be filled with uniform fluid of the ſame denſity: the force with which a particle of fluid P, placed anywhere at pleaſure, repels the whole quantity of fluid in AF, in the direction of the canal, is the ſame with which it repels the fluid in the canal AE, in the direction AE.

On the center P, draw two circular arches BD and *bd*, infinitely near to each other, cutting AE in B and β, and AF in D and δ, and draw the radii P*b* and P*d*. As $PB = PD$, the force with which P repels a particle at B, in the direction $B\beta$, is to that with which it repels an equal particle at D, in the direction $D\delta$, as $\frac{Bb}{B\beta}$ to $\frac{Dd}{D\delta}$, or as $\frac{1}{B\beta}$ to $\frac{1}{D\delta}$; and therefore, the force with which it repels the whole fluid in $B\beta$, in the direction $B\beta$, is the ſame with which it repels the whole fluid in $D\delta$, in the direction $D\delta$, that is in the direction of the canal; and therefore, the force with which it repels the whole fluid in AE, in the direction AE, is the ſame with which it repels the whole fluid in AF, in the direction of the canal.

COROLLARY.

If the bent canal ADF, inſtead of being infinitely continued, meets the cylindric canal in E, as in fig. 16. the repulſion of P on the fluid in the bent canal A D E, in the direction of the canal, will ſtill be equal to its repulſion on that in the cylindric canal A E, in the direction A E.

PROP. XXIV.

If two bodies, for inſtance the plate A B, and the body H, of Prop. XXII. communicate with each other, by a canal filled with incompreſſible fluid, and are either over or undercharged, the quantity of redundant fluid in them will bear the ſame proportion to each other, whether the canal by which they communicate is ſtraight or crooked, or into whatever part of the bodies the canal is inſerted, or in whatever manner the two bodies are ſituated in reſpect of each other; provided that their diſtance is infinite, or ſo great that the repulſion of each body on the fluid in the canal ſhall not be ſenſibly leſs than if it was infinite.

Let the parellelograms AB and DF (fig. 17.) repreſent the two plates, and H and L the bodies communicating with them: let now H be removed to *h*; and let it communicate with AB, by the bent canal *g c*; the quantity of fluid in the plates and bodies remaining the ſame as before; and let us, for the ſake of eaſe in the demonſtration, ſuppoſe the canal *g c* to be every where of the ſame thickneſs as the canal G C; though the propoſition will evidently hold good equally, whether it is or not: the fluid will ſtill be in equilibrio. For let us firſt ſuppoſe the canal *g c* to be continued through the ſubſtance of the plate A B, to C, along the line *c r* C; the part *c r* C being of the ſame thickneſs as the reſt of the canal, and the fluid in it of the ſame denſity: by the preceding propoſition, the repulſion or attraction of each particle of fluid or matter in the plates A B and D F, on the fluid in the whole canal C *r c g*, in the direction of that canal, is equal to its repulſion or attraction on the fluid in the canal C G, in the direction C G; and therefore the whole repulſion or attraction of the two plates on the canal C *r c g*, is equal to their repulſion or attraction on C G: but as the fluid in the plate A B is in equilibrio, each particle of fluid in the part C *r c* of the canal, is impelled by the plates, with as much force in one direction as the other; and conſequently the plates impel the fluid in the canal *c g*, with as much force as they do that in the whole canal C *r c g*, that is, with the ſame force that they impel the fluid in C G. In like manner the body *b* impels the fluid in *c g*, with the ſame force that H does the fluid in C G; and conſequently *b* impels the fluid in *c g*, one way in the direction of the canal, with the ſame force that the two plates impel it the contrary way; and therefore the fluid in *c g* has no tendency to flow from one body to the other.

COROLLARY.

By the ſame method of reaſoning, with the help of the corollary to the 23d propoſition, it appears, that if A B and H each communicate with a third body, by canals of incompreſſible fluid, and a communication is made between A B and H by another canal of incompreſſible fluid, the fluid will have no tendency to flow from one to the other through this canal; ſuppoſing that the fluid was in equilibrio before this communication was made. In like manner if A B and H communicate with each other, or each communicate with a third body, by canals of real fluid, inſtead of the imaginary canals of incompreſſible fluid uſed in theſe propoſitions, and a communication is alſo made between them by a canal of incompreſſible fluid, the fluid can have no tendency to flow from one to the other. The truth of the latter part of this corollary will appear by ſuppoſing an imaginary canal of incompreſſible fluid to be continued through the whole length of the real one.

PROP. XXV.

Let now a communication be made between the two plates AB and DF, by the canal NRS of incompreſſible fluid, of any length; and let the body H and the plate AB be overcharged. It is plain that the fluid will flow through that canal from A B to D F. Now the whole force with which the fluid in the canal is impelled

along

along it, by the joint action of the two plates, is the ſame with which the whole quantity of fluid in the canal CG or *cg* is impelled by them; ſuppoſing the canal N R S to be every where of the ſame breadth and thickneſs as CG or *cg*.

For ſuppoſe that the canal N R S, inſtead of communicating with the plate DF, is bent back juſt before it touches it, and continued infinitely along the line S*s*; the force with which the two plates impel the fluid in S*s*, is the ſame with which they impel that in EL, ſuppoſing S *s* to be of the ſame breadth and thickneſs as EL; and is therefore nothing; therefore the force with which they impel the fluid in NRS, is the ſame with which they impel that in N R S *s*; which is the ſame with which they impel that in CG.

PROP. XXVI.

Let now *xyz* be a body of an infinite ſize, containing juſt fluid enough to ſaturate it; and let a communication be made between *b* and *xyz*, by the canal *by* of incompreſſible fluid, of the ſame breadth and thickneſs as *gc* or GC; the fluid will flow through it from *b* to *xyz*; and the force with which the fluid in that canal is impelled along it, is equal to that with which the fluid in NRS is impelled by the two plates.

If the canal *by* is of ſo great a length, that the repulſion of *b* thereon is the ſame as if it was continued infinitely, then the thing is evident: but if it is not, let the canal *by*, inſtead of communicating with *xyz*, ſo that the fluid can flow out of the canal into *xyz*, be continued infinitely through its ſubſtance, along the line *yv*: now it muſt be obſerved that a ſmall part of the body *xyz*, namely, that which is turned towards *b*, will by the action of *b* upon it, be rendered undercharged; but all the reſt of the body will be ſaturated; for the fluid driven out of the undercharged part will not make the remainder, which is ſuppoſed to be of an infinite ſize, ſenſibly overcharged: now the force with which the fluid in the infinite canal *byv*, is impelled by the body *b* and the undercharged part of *xyz*, is the ſame with which the fluid in *gc* is impelled by them; but as the fluid in all parts of *xyz* is in equilibrio, a particle in any part of *yv* cannot be impelled in any direction; and therefore the fluid in *by* is impelled with as much force as that in *byv*; and therefore the fluid in *by* is impelled with as much force as that in *gc*; and is therefore impelled with as much force as the fluid in NRS is impelled by the two plates.

It perhaps may be aſked, whether this method of demonſtration would not equally tend to prove that the fluid in *by* was impelled with the ſame force as that in NRS, though *xyz* did not contain juſt fluid enough to ſaturate it. I anſwer not; for this demonſtration depends on the canal *yv* being continued, within the body *xyz*, to an infinite diſtance beyond any over or undercharged part; which could not be if xyz contained either more or leſs fluid than that.

PROP. XXVII.

Let two bodies B and *b* (fig. 13.) be joined by a cylindric or priſmatic canal A *a*, filled with real fluid; and not by an imaginary canal of incompreſſible fluid as in the 20th propoſition; and let the fluid therein be in equilibrio: the force with which the whole or any given part of the fluid in the canal, is impelled in the direction of its axis, by the united repulſions and attractions of the redundant fluid or matter in the two bodies and the canal, muſt be nothing; or the force with which it is impelled one way in the direction of the axis of the canal, muſt be equal to that with which it is impelled the other way.

For as the canal is ſuppoſed cylindric or priſmatic, no particle of fluid therein can be prevented from moving in the direction of the axis of it, by the ſides of the canal; and therefore the force with which each particle is impelled either way in the direction of the axis, by the united attractions and repulſions of the two bodies and the canal, muſt be nothing, otherwiſe it could not be at reſt; and therefore the force with which the whole, or any given part of the fluid in the canal, is impelled in the direction of the axis, muſt be nothing.

COROL. I.

If the fluid in the canal is diſpoſed in ſuch manner, that the repulſion or attraction of the redundant fluid or matter in it, on the whole or any given part of the fluid in the canal, has no tendency to impel it either way in the direction of the axis; then the force with which that whole or given part is impelled by the two bodies muſt be nothing; or the force with which it is impelled one way in the direction of the axis, by the body B, muſt be equal to that with which it is impelled in the contrary direction by the other body; but not if the fluid in the canal is diſpoſed in a different manner.

COROL. II.

If the bodies, and conſequently the canal, is overcharged; then, in whatever manner the fluid in the canal is diſpoſed, the force with which the whole quantity of redundant fluid in the canal is repelled by the body B in the direction A *a*, muſt be equal to that with which it is repelled by *b* in the contrary direction. For the force with which the redundant fluid is impelled in the direction A *a* by its own repulſion, is nothing; for the repulſion of the particles of any body on each other have no tendency to make the whole body move in any direction.

REMARKS.

When I firſt thought of the 20th and 22d propoſitions, I imagined that when two bodies were connected by a cylindric canal of real fluid, the repulſion of one body on the whole quantity of fluid in the canal, in one direction, would be equal to that of the other body on it in the contrary direction, in whatever manner the fluid was diſpoſed in the canal; and that therefore thoſe propoſitions would have held good very nearly, though the bodies were joined by cylindric canals of real fluid; provided the bodies were ſo little over or undercharged, that the quantity of redundant or deficient fluid in the canal ſhould be very ſmall in reſpect of the quantity required to ſaturate it; and conſequently that the fluid therein ſhould be very nearly of the ſame denſity in all parts. But from the foregoing propoſition it appears that I was miſtaken, and that the repulſion of one body on the fluid in the canal is not equal to that of the other body on it, unleſs the fluid in the canal is diſpoſed in a particular manner: beſides that, when two bodies are both joined by a real canal, the attraction or repulſion of the redundant matter or fluid in the canal, has ſome tendency to alter the diſpoſition of the fluid in the two bodies; and in the 22d propoſition, the canal C G exerts alſo ſome attraction or repulſion on the canal E M: on all which accounts the demonſtration of thoſe propoſitions is defective, when the bodies are joined by real canals. I have good reaſon however to think, that thoſe propoſitions actually hold good very nearly when the bodies are joined by real canals; and that, whether the canals are ſtraight or crooked, or in whatever direction the bodies are ſituated in reſpect of each other: though I am by no means able to prove that they do: I therefore choſe ſtill to retain thoſe propoſitions, but to demonſtrate them on this ideal ſuppoſition, in which they are certainly true, in hopes that ſome more ſkilful mathematician may be able to ſhew whether they really hold good or not.

What principally makes me think that this is the caſe, is that as far as I can judge from ſome experiments I have made, the quantity of fluid in different bodies agrees very well with thoſe propoſitions, on a ſuppoſition that the electric repulſion is inverſely as the ſquare of the diſtance. It ſhould alſo ſeem from thoſe experiments, that the quantity of redundant or deficient fluid in two bodies, bore very nearly the ſame proportion to each other, whatever is the ſhape of the canal by which they are joined, or in whatever direction they are ſituated in reſpect of each other.

Though the above propoſitions ſhould be found not to hold good, when the bodies are joined by real canals, ſtill it is evident, that in the 22d propoſition, if the plates AB and DF are very near together, the quantity of redundant fluid in the plate AB will be many times greater than that in the body H, ſuppoſing H to conſiſt of a circular plate of the ſame ſize as AB, and DF will be near as much undercharged as AB is overcharged.

Sir Iſaac Newton ſuppoſes that air conſiſts of particles which repel each other with a force inverſely as the diſtance: but it appears plainly from the foregoing pages, that if the repulſion of the particles was in this ratio; and extended indefinitely to all diſtances, they would compoſe a fluid extremely different from common air. If the repulſion of the particles was inverſely as the diſtance, but extended only to a given very ſmall diſtance from their centers, they would compoſe a fluid of the ſame kind as air, in reſpect of elaſticity, except that its denſity would not be in proportion to its compreſſion: if the diſtance to which the repulſion extends, though very ſmall, is yet many times greater than the diſtance of the particles from each other, it might be ſhewn, that the denſity of the fluid would be nearly as the ſquare root of the compreſſion. If the repulſion of the particles extended indefinitely, and was inverſely as ſome higher power of the diſtance than the cube, the denſity of the fluid would be as ſome power of the compreſſion leſs than $\frac{3}{5}$. The only law of repulſion, I can think of, which will agree with experiment, is one which ſeems not very likely; namely, that the particles repel each other with a force inverſely as the diſtance; but that, whether the denſity of the fluid is great or ſmall, the repulſion extends only to the neareſt particles: or, what comes to the ſame thing, that the diſtance to which the repulſion extends, is very ſmall, and alſo is not fixed, but varies in proportion to the diſtance of the particles.

PART II.

Containing a compariſon of the foregoing theory with experiment.

§ 1. It appears from experiment, that ſome bodies ſuffer the electric fluid to paſs with great readineſs between their pores; while others will not ſuffer it to do ſo without great difficulty; and ſome hardly ſuffer it to do ſo at all. The firſt ſort of bodies are called conductors, the others non-conductors. What this difference in bodies is owing to I do not pretend to explain.

It is evident that the electric fluid in non-conductors may be conſidered as moveable, or anſwers to the definition given of that term in p. 588. As to the fluid contained in non-conducting ſubſtances, though it does not abſolutely anſwer to the definition of immoveable, as it is not abſolutely confined from moving, but only does ſo with great difficulty; yet it may in moſt caſes be looked upon as ſuch without ſenſible error.

Air does in ſome meaſure permit the electric fluid to paſs through it; though, if it is dry, it lets it paſs but very ſlowly, and not without difficulty; it is therefore to be called a non-conductor.

It appears that conductors would readily ſuffer the fluid to run in and out of them, were it not for the air which ſurrounds them: for if the end of a conductor is inſerted into a vacuum, the fluid runs in and out of it with perfect readineſs; but when it is ſurrounded on all ſides by the air, as no fluid can run out of it without running into the air, the fluid will not do ſo without difficulty.

If any body is ſurrounded on all ſides by the air, or other non-conducting ſubſtances, it is ſaid to be inſulated: if on the other hand it any where communicates with any conducting body, it is ſaid to be not inſulated. When I ſay that a body communicates with the ground, or any other body, I would be underſtood to mean that it does ſo by ſome conducting ſubſtance.

Though the terms poſitively and negatively electrified are much uſed, yet the preciſe ſenſe in which they are to be underſtood, ſeems not well aſcertained; namely, whether they are to be underſtood in the ſame ſenſe in which I have uſed the words over or undercharged, or whether, when any number of bodies, inſulated and communicating with each other by conducting ſubſtances, are electrified by means of excited glaſs, they are all to be called poſitively electrified (ſuppoſing, according to the uſual opinion, that excited glaſs contains more than its natural quantity of electricity); even though ſome of them, by the approach of a ſtronger electrified body, are made undercharged. I ſhall uſe the words in the latter ſenſe; but as it will be proper to aſcertain the ſenſe in which I ſhall uſe them more accurately, I ſhall give the following definition.

In order to judge whether any body, as A, is poſitively or negatively electrified: ſuppoſe another body B, of a given ſhape and ſize, to be placed at an infinite diſtance from it, and from any other over or undercharged body; and let B contain the ſame quantity of electric fluid, as if it communicated with A by a canal of incompreſſible fluid: then, if B is overcharged, I call A poſitively electrified; and if it is undercharged, I call A negatively electrified; and the greater the degree in which B is over or undercharged, the greater is the degree in which A is poſitively or negatively electrified.

It appears from the corollary to the 24th propoſition, that if ſeveral bodies are inſulated, and connected together by conducting ſubſtances, and one of theſe bodies is poſitively or negatively electrified, all the other bodies muſt be electrified in the ſame degree: for ſuppoſing a given body B to be placed at an infinite diſtance from any over or undercharged body, and to contain the ſame quantity of fluid as if it communicated with one of thoſe bodies by a canal of incompreſſible fluid, all the reſt of thoſe bodies muſt by that corollary contain the ſame quantity of fluid as if they communicated with B by canals of incompreſſible fluid: but yet it is poſſible that ſome of thoſe bodies may be overcharged, and others undercharged: for ſuppoſe the bodies to be poſitively electrified, and let an overcharged body D be brought near one of them, that body will become undercharged, provided D is ſufficiently overcharged; and yet by the definition it will ſtill be poſitively electrified in the ſame degree as before.

Moreover, if ſeveral bodies are inſulated and connected together by conducting ſubſtances, and one of theſe bodies is electrified by excited glaſs, there can be no doubt, I think, but what they will all be poſitively electrified; for if there is no other over or undercharged body placed near any of theſe bodies, the thing is evident; and though ſome of theſe bodies may, by the approach of a ſufficiently overcharged body, be rendered undercharged; yet I do not ſee how it is poſſible to prevent a body placed at an infinite diſtance, and communicating with them by a canal of incompreſſible fluid, from being overcharged.

In like manner if one of theſe bodies is electrified by excited ſealing wax, they will all be negatively electrified.

It is impoſſible for any body communicating with the ground to be either poſitively or negatively electrified: for the earth, taking the whole together, contains juſt fluid enough to ſaturate it, and conſiſts in general of conducting ſubſtances; and conſequently though it is poſſible for ſmall parts of the ſurface of the earth to be rendered over or undercharged, by the approach of electrified clouds or other cauſes; yet the bulk of the earth, and eſpecially the interior parts, muſt be ſaturated with electricity. Therefore aſſume any part of the earth which is itſelf ſaturated, and is at a great diſtance from any over or undercharged part; any body communicating with the ground, contains as much electricity as if it communicated with this part by a canal of incompreſſible fluid, and therefore is not at all electrified.

If any body A, inſulated and ſaturated with electricity, is placed at a great diſtance from any over or undercharged body, it is plain that it cannot be electrified; but if an overcharged body is brought near it, it will be poſitively electrified; for ſuppoſing A to communicate with any body B, at an infinite diſtance, by a canal of incompreſſible fluid, it is plain that unleſs B is overcharged, the fluid in the canal could not be in equilibrio, but would run from A to B. For the ſame reaſon a body inſulated and ſaturated with fluid, will be negatively electrified if placed near an undercharged body.

§ 2. The phænomena of the attraction and repulſion of electrified bodies ſeem to agree exactly with the theory; as will appear by conſidering the following caſes.

CASE I. Let two bodies, A and B, both conductors of electricity, and both placed at a great diſtance from any other electrified bodies, be brought near each other. Let A be inſulated, and contain juſt fluid enough to ſaturate it; and let B be poſitively electrified. They will attract each other; for as B is poſitively electrified, and at a great diſtance from any overcharged body, it will be overcharged; therefore, on approaching A and B to each other, ſome fluid will be driven from that part of A which is neareſt to B to the further part: but when the fluid in A was ſpread uniformly, the repulſion of B on the fluid in A was equal to its attraction on the matter therein; therefore, when ſome fluid is removed from thoſe parts where the repulſion of B is ſtrongeſt to thoſe where it is weaker, B will repel the fluid in A with leſs force than it attracts the matter; and conſequently the bodies will attract each other.

Case II. If we now ſuppoſe that the fluid is at liberty to eſcape from out of A, if it has any diſpoſition to do ſo, the quantity of fluid in it before the approach of B being ſtill ſufficient to ſaturate it; that is, if A is not inſulated and not electrified, B being ſtill poſitively electrified, they will attract with more force than before: for in this caſe, not only ſome fluid will be driven from that part of A which is neareſt to B to the oppoſite part, but alſo ſome fluid will be driven out of A.

It muſt be obſerved, that if the repulſion of B on a particle at E, (fig. 19.) the fartheſt part of A, is very ſmall in reſpect of its repulſion on an equal particle placed at D, the neareſt part of A, the two bodies will attract with very nearly the ſame force, whether A is inſulated or not; but if the repulſion of B, on a particle at E, is very near as great as on one at D, they will attract with very little force if A is inſulated. For inſtance, let a ſmall overcharged ball be brought near one end of a long conductor not electrified; they will attract with very near the ſame force, whether the conductor be inſulated or not; but if the conductor be overcharged, and brought near a ſmall unelectrified ball, they will not attract with near ſo much force, if the ball is inſulated, as if it is not.

Case III. If we now ſuppoſe that A is negatively electrified, and not inſulated, it is plain that they will attract with more force than in the laſt caſe; as A will be ſtill more undercharged in this caſe, than in the laſt.

N. B. In theſe three caſes, we have not as yet taken notice of the effect which the body A will have in altering the quantity and diſpoſition of the fluid in B; but in reality this will make the bodies attract each other with more force than they would otherwiſe do; for in each of theſe caſes the body A attracts the fluid in B; which will cauſe ſome fluid to flow from the farther parts of B to the nearer, and will alſo cauſe ſome fluid to flow into it, if it is not inſulated, and will conſequently cauſe B to act upon A with more force than it would otherwiſe do.

Case IV. Let us now ſuppoſe that B is negatively electrified; and let A be inſulated, and contain juſt fluid enough to ſaturate it; they will attract each other; for B will be undercharged; it will therefore attract the fluid in A, and will cauſe ſome fluid to flow from the fartheſt part of A, where it is attracted with leſs force, to the nearer part, where it is attracted with more force; ſo that B will attract the fluid in A with more force than it repels the matter.

Case V, and VI. If A is now ſuppoſed to be not inſulated and not electrified, B being ſtill negatively electrified, it is plain that they will attract with more force than in the laſt caſe: and if A is poſitively electrified, they will attract with ſtill more force.

In theſe three laſt caſes alſo, the effect which A has in altering the quantity and diſpoſition of the fluid in B, tends to increaſe the force with which the two bodies attract.

CASE VII. It is plain that a non-conducting body ſaturated with fluid, is not at all attracted or repelled by an over or undercharged body, until, by the action of the electrified body on it, it has either acquired ſome additional fluid from the air, or had ſome driven out of it, or till ſome fluid is driven from one part of the body to the other.

CASE VIII. Let us now ſuppoſe that the two bodies A and B are both poſitively electrified in the ſame degree. It is plain, that were it not for the action of one body on the other, they would both be overcharged, and would repel each other. But it may perhaps be ſaid, that one of them as A may, by the action of the other on it, be either rendered undercharged on the whole, or at leaſt may be rendered undercharged in that part neareſt to B; and that the attraction of this undercharged part on a particle of the fluid in B, may be greater than the repulſion of the more diſtant overcharged part; ſo that on the whole the body A may attract a particle of fluid in B. If ſo, it muſt be affirmed that the body B repels the fluid in A; for otherwiſe, that part of A which is neareſt to B could not be rendered undercharged. Therefore, to obviate this objection, let the bodies be joined by the ſtraight canal DC of incompreſſible fluid (fig. 19.). The body B will repel the fluid in all parts of this canal; for as A is ſuppoſed to attract the fluid in B, B will not only be more overcharged than it would otherwiſe be, but it will alſo be more overcharged in that part neareſt to A than in the oppoſite part. Moreover, as the near undercharged part of A is ſuppoſed to attract a particle of fluid in B with more force than the more diſtant overcharged part repels it; it muſt, *a fortiori*, attract a particle in the canal with more force than the other repels it; therefore the body A muſt attract the fluid in the canal; and conſequently ſome fluid muſt flow from B to A, which is impoſſible; for as A and B are both electrified in the ſame degree, they contain the ſame quantity of fluid as if they both communicated with a third body at an infinite diſtance, by canals of incompreſſible fluid; and therefore, by the corollary to Prop. 24, if a communication is made between them by a canal of incompreſſible fluid, the fluid would have no diſpoſition to flow from one to the other.

CASE IX. But if one of the bodies as A is poſitively electrified in a leſs degree than B, then it is poſſible for the bodies to attract each other; for in this caſe the force with which B repels the fluid in A may be ſo great, as to make the body A either intirely undercharged, or at leaſt to make the neareſt part of it ſo much undercharged, that A ſhall on the whole attract a particle of fluid in B.

It may be worth remarking with regard to this caſe, that when two bodies, both electrified poſitively but unequally, attract each other, you may by removing them to a greater diſtance from each other, cauſe them to repel; for as the ſtronger electrified body repels the fluid in the weaker with leſs force when removed to a greater diſtance, it will not be able to drive ſo much fluid out of it, or from the nearer to the further part, as when placed at a leſs diſtance.

CASE X, and XI. By the ſame reaſoning it appears, that if the two bodies are both negatively electrified in the ſame degree, they muſt repel each other: but if they are both negatively electrified in different degrees, it is poſſible for them to attract each other.

All theſe caſes are exactly conformable to experiment.

CASE XII. Let two cork balls be ſuſpended by conducting threads from the ſame poſitively electrified body, in ſuch manner that if they did not repel, they would hang cloſe together: they will both be equally electrified, and will repel each other: let now an overcharged body, more ſtrongly electrified than them, be brought under them; they will become leſs overcharged, and will ſeparate leſs than before: on bringing the body ſtill nearer, they will become not at all overcharged, and will not ſeparate at all: and on bringing the body ſtill nearer, they will become undercharged, and will ſeparate again.

CASE XIII. Let all the air of a room be overcharged, and let two cork balls be ſuſpended cloſe to each other by conducting threads communicating with the wall. By Prop. 15, it is highly probable that the balls will be undercharged; and therefore they ſhould repel each other.

Theſe two laſt caſes are experiments of Mr. Canton's, and are deſcribed in Philoſ. Tranſ. 1753, p. 350, where are other experiments of the ſame kind, all readily explicable by the foregoing theory.

I have now conſidered all the principal or fundamental caſes of electric attractions and repulſions which I can think of; all of which appear to agree perfectly with the theory.

§ 3. On the caſes in which bodies receive electricity from or part with it to the air.

LEMMA I.

Let the body A (fig. 6.) either ſtand near ſome over or undercharged body, or at a diſtance from any. It ſeems highly probable, that if any part of its ſurface, as MN, is overcharged, the fluid will endeavour to run out through that part, provided the air adjacent thereto is not overcharged.

For let G be any point in that ſurface, and P a point within the body, extremely near to it; it is plain that a particle of fluid at P, muſt be repelled with as much force in one direction as another (otherwiſe it could not be at reſt) unleſs all the fluid between P and G is preſſed cloſe together, in which caſe it may be repelled with more force towards G than it is in the contrary direction: now a particle at G is repelled in the direction PG, *i. e.* from P to G, by all the redundant fluid between P and G; and a particle at P is repelled by the ſame fluid in the contrary direction; ſo that as the particle at P is repelled with not leſs force in the direction PG than in the

contrary,

contrary, I do not ſee how a particle at G can help being repelled with more force in that direction than the contrary, unleſs the air on the outſide of the ſurface M N was more overcharged than the ſpace between P and G.

In like manner, if any part of the ſurface is undercharged, the fluid will have a tendency to run in at that part from the air.

The truth of this is ſomewhat confirmed by the third problem; as in all the caſes of that problem, the fluid was ſhewn to have a tendency to run out of the ſpaces A D and E H, at any ſurface which was overcharged, and to run in at any which was undercharged.

COROL. I.

If any body at a diſtance from other over or undercharged bodies, be poſitively electrified, the fluid will gradually run out of it from all parts of its ſurface into the adjoining air; as it is plain that all parts of the ſurface of that body will be overcharged: and if the body is negatively electrified, the fluid will gradually run into it at all parts of its ſurface from the adjoining air.

COROL. II.

Let the body A (fig. 6.) inſulated and containing juſt fluid enough to ſaturate it, be brought near the overcharged body B; that part of the ſurface of A which is turned towards B will by Prcp. 11. be rendered undercharged, and will therefore imbibe electricity from the air; and at the oppoſite ſurface R S, the fluid will run out of the body into the air.

COROL. III.

If we now ſuppoſe that A is not inſulated, but communicates with the ground, and conſequently that it contained juſt fluid enough to ſaturate it before the approach of B, it is plain that the ſurface M N will be more undercharged than before; and therefore the fluid will run in there with more force than before; but it can hardly have any diſpoſition to run out at the oppoſite ſurface R S; for if the canal by which A communicates with the ground is placed oppoſite to B, as in figure 5, then the fluid will run out through that canal till it has no longer any tendency to run out at R S; and by the remarks at the end of Prop. 27, it ſeems probable, that the fluid in A will be nearly in the ſame quantity, and diſpoſed nearly in the ſame manner, into whatever part of A the canal is inſerted by which it communicates with the ground.

COROL. IV.

If B is undercharged the caſe will be reverſed; that is, it will run out where it before run in, and will run in where it before run out.

As far as I can judge, theſe corollaries ſeem conformable to experiment: thus far is certain, that bodies at a diſtance from other electrified bodies receive electricity from the air, if negatively electrified, and part with ſome to it if poſitively electrified: and a body not electrified and not inſulated receives electricity from the air if brought near an overcharged body, and loſes ſome when brought near an undercharged body: and a body inſulated and containing its natural quantity of fluid, in ſome caſes, receives, and in others loſes electricity, when brought near an over or undercharged body.

§ 4. The well-known effects of points in cauſing a quick diſcharge of electricity ſeem to agree very well with this theory.

It appears from the 20th propoſition, that if two ſimilar bodies of different ſizes are placed at a very great diſtance from each other, and connected by a ſlender canal, and overcharged, the force with which a particle of fluid placed cloſe to correſponding parts of their ſurface is repelled from them, is inverſely as the correſponding diameters of the bodies. If the diſtance of the two bodies is ſmall, there is not ſo much difference in the force with which the particle is repelled by the two bodies; but ſtill, if the diameters of the two bodies are very different, the particle will be repelled with much more force from the ſmaller body than from the larger. It is true indeed that a particle placed at a certain diſtance from the ſmaller body, will be repelled with leſs force than if it be placed at the ſame diſtance from the greater body; but this diſtance is, I believe, in moſt caſes pretty conſiderable; if the bodies are ſpherical, and the repulſion inverſely as the ſquare of the diſtance, a particle placed at any diſtance from the ſurface of the ſmaller body leſs than a mean proportional between the radii of the two bodies, will be repelled from it with more force than if it be placed at the ſame diſtance from the larger body.

I think therefore that we may be well aſſured that if two ſimilar bodies are connected together by a ſlender canal, and are overcharged, the fluid muſt eſcape faſter from the ſmaller body than from an equal ſurface of the larger; but as the ſurface of the larger body is greateſt, I do not know which body ought to loſe moſt electricity in the ſame time; and indeed it ſeems impoſſible to determine poſitively from this theory which ſhould, as it depends in great meaſure on the manner in which the air oppoſes the entrance of the electric fluid into it. Perhaps in ſome degrees of electrification the ſmaller body may loſe moſt, and in others the larger.

Let now ACB (fig. 18.) be a conical point ſtanding on any body DAB, C being the vertex of the cone; and let DAB be overcharged: I imagine that a particle of fluid placed cloſe to the ſurface of the cone anywhere between *b* and C, muſt be repelled with at leaſt as much, if not more, force than it would, if the part A*ab*B of the cone was taken away, and the part *a*C*b* connected to DAB by a ſlender canal; and conſequently, from what has been ſaid before, it ſeems reaſonable to ſuppoſe that the waſte of electricity from the end of the cone muſt be very great in proportion to its ſurface; though it does not appear from this reaſoning whether the waſte of electricity from the whole cone ſhould be greater or leſs than from a cylinder of the ſame baſe and altitude.

All which has been here ſaid relating to the flowing out of electricity from overcharged bodies, holds equally true with regard to the flowing in of electricity into undercharged bodies.

But a circumſtance which I believe contributes as much as any thing to the quick diſcharge of electricity from points, is the ſwift current of air cauſed by them, and taken notice of by Mr. Wilſon and Dr. Prieſtly (*vide* Prieſtly, p. 117 and 591); and which is produced in this manner.

If a globular body A B D is overcharged, the air cloſe to it, all round its ſurface, is rendered overcharged, by the electric fluid, which flows into it from the body; it will therefore be repelled by the body; but as the air all round the body is repelled with the ſame force, it is in equilibrio, and has no tendency to fly off from it. If now the conical point A C B be made to ſtand out from the globe, as the fluid will eſcape much faſter in proportion to the ſurface from the end of the point than from the reſt of the body, the air cloſe to it will be much more overcharged than that cloſe to the reſt of the body; it will therefore be repelled with much more force; and conſequently a current of air will flow along the ſides of the cone, from B towards C; by which means there is a continual ſupply of freſh air, not much overcharged, brought in contact with the point; whereas otherwiſe the air adjoining to it would be ſo much overcharged, that the electricity would have but little diſpoſition to flow from the point into it.

The ſame current of air is produced in a leſs degree, without the help of the point, if the body, inſtead of being globular, is oblong or flat, or has knobs on it, or is otherwiſe formed in ſuch manner as to make the electricity eſcape faſter from ſome parts of it than the reſt.

In like manner, if the body ABD be undercharged, the air adjoining to it will alſo be undercharged, and will therefore be repelled by it; but as the air cloſe to the end of the point will be more undercharged than that cloſe to the reſt of the body, it will be repelled with much more force; which will cauſe exactly the ſame current of air, flowing the ſame way, as if the body was overcharged; and conſequently the velocity with which the electric fluid flows into the body, will be very much increaſed. I believe indeed that it may be laid down as a conſtant rule, that the faſter the electric fluid eſcapes from any body when overcharged, the faſter will it run into that body when undercharged.

Points are not the only bodies which cauſe a quick diſcharge of electricity; in particular, it eſcapes very faſt from the ends of long ſlender cylinders; and a ſwift current of air is cauſed to flow from the middle of the cylinder towards the end: this will eaſily appear by conſidering that the redundant fluid is collected in much greater quantity near the ends of the cylinders than near the middle. The ſame thing may be ſaid, but I believe in a leſs degree, of the edges of thin plates.

What has been juſt ſaid concerning the current of air, ſerves to explain the reaſon of the revolving motion of Dr. Hamilton's and Mr. Kinnerſley's bent pointed wires, vide Phil. Tranſ. vol. LI, p. 905, and vol. LIII, p. 86; alſo Prieſtly, p. 429: for the ſame repulſion which impels the air from the thick part of the wire towards the point, tends to impel the wire in the contrary direction.

It is well known, that if a body B is poſitively electrified, and another body A, communicating with the ground, be then brought near it, the electric fluid will eſcape faſter from B, at that part of it which is turned towards A, than before. This is plainly conformable to theory; for as A is thereby rendered undercharged, B will in its turn be made more overcharged, in that part of it which is turned towards A, than it was before. But it is alſo well known that the fluid will eſcape faſter from B, if A be pointed, than if it be blunt; though B will be leſs overcharged in this caſe than in the other; for the broader the ſurface of A, which is turned towards B, the more effect will it have in increaſing the overcharge of B. The cauſe of this phænomenon is as follows:

If A is pointed, and the pointed end turned towards B, the air cloſe to the point will be very much undercharged, and therefore will be ſtrongly repelled by A, and attracted by B, which will cauſe a ſwift current of air to flow from it towards B; by which means a conſtant ſupply of undercharged air will be brought in contact with B, which will accelerate the diſcharge of electricity from it in a very great degree: and moreover, the more pointed A is, the ſwifter will be this current. If, on the other hand, that end of A which is turned towards B, is ſo blunt, that the electricity is not diſpoſed to run into A faſter than it is to run out of B, the air adjoining to B may be as much overcharged as that adjoining to A is undercharged; and therefore may by the joint repulſion of B and attraction of A, be impelled from B to A, with as much or more force than the air adjoining to A is impelled in the contrary direction; ſo that what little current of air there is may flow in the contrary direction.

It is eaſy applying what has been here ſaid to the caſe in which B is negatively electrified.

§ 5. In the paper of Mr. Canton's, quoted in the ſecond ſection, and in a paper of Dr. Franklin's (Phil. Tranſ. 1755, p. 300, and Franklin's letters p. 155.) are ſome remarkable experiments, ſhewing that when an overcharged body is brought near another body, ſome fluid is driven to the further end of this body, and alſo ſome driven out of it, if it is not inſulated. The experiments are all ſtrictly conformable to the 11th, 12th, and 13th propoſitions: but it is needleſs to point out the agreement, as the explanation given by the authors does it ſufficiently.

§ 6. On the Leyden vial.

The ſhock produced by the Leyden vial ſeems owing only to the great quantity of redundant fluid collected on its poſitive ſide, and the great deficiency on its negative ſide; ſo that if a conductor was prepared of ſo great a ſize, as to be able to receive as much additional fluid by the ſame degree of electrification as the poſitive ſide of a Leyden vial, and was poſitively electrified in the ſame degree as the vial, I do not doubt but what as great a ſhock would be produced by making a communication between this conductor and the ground, as between the two ſurfaces of the

Leyden vial, ſuppoſing both communications to be made by canals of the ſame length and ſame kind.

It appears plainly from the experiments which have been made on this ſubject, that the electric fluid is not able to paſs through the glaſs; but yet it ſeems as if it was able to penetrate without much difficulty to a certain ſmall depth, perhaps I might ſay an imperceptible depth within the glaſs; as Dr. Franklin's analyſis of the Leyden vial ſhews that its electricity is contained chiefly in the glaſs itſelf, and that the coating is not greatly over or undercharged.

It is well known that glaſs is not the only ſubſtance which can be charged in the manner of the Leyden vial; but that the ſame effect may be produced by any other body, which will not ſuffer the electricity to paſs through it.

* Hence the phænomena of the vial ſeem eaſily explicable by means of the 22d propoſition. For let ACGM, fig. 20, repreſent a flat plate of glaſs or any other ſubſtance which will not ſuffer the electric fluid to paſs through it, ſeen edgeways; and let B*bd*D, and E*ef*F, or B*d* and E*f*, as I ſhall call them for ſhortneſs, be two plates of conducting matter of the ſame ſize, placed in contact with the glaſs oppoſite to each other; and let B*d* be poſitively electrified; and let E*f* communicate with the ground; and let the fluid be ſuppoſed either

able to enter a little way into the glaſs, but not to paſs through it, or unable to enter it at all; and if it is able to enter a little way into it, let $b\beta\delta d$, or $b\delta$, as I ſhall call it, repreſent that part of the glaſs into which the fluid can enter from the plate Bd, and $e\varphi$, that which the fluid from Ef can enter. By the abovementioned propoſition, if be, the thickneſs of the glaſs, is very ſmall in reſpect of bd, the diameter of the plates, the quantity of redundant fluid forced into the ſpace Bd, or $B\delta$, (that is, into the plate Bd, if the fluid is unable to penetrate at all into the glaſs, or into the plate Bd, and the ſpace $b\delta$ together, if the fluid is able to penetrate into the glaſs) will be many times greater than what would be forced into it by the ſame degree of electrification if it had been placed by itſelf; and the quantity of fluid driven out of $E\varphi$, will be nearly equal to the redundant fluid in $B\delta$.

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* The following explication is ſtrictly applicable only to that ſort of Leyden vial, which conſiſts of a flat plate of glaſs or other matter. It is evident, however, that the reſult muſt be nearly of the ſame kind, though the glaſs is made into the ſhape of a bottle as uſual, or into any other form: but I propoſe to conſider thoſe ſort of Leyden vials more particularly in a future paper.

If a communication be now made between $B\delta$ and $E\varphi$, by the canal NRS, the redundant fluid will run from $B\delta$ to $E\varphi$; and if in its way it paſſes through the body of any animal, it will by the rapidity of its motion produce in it that ſenſation called a ſhock.

It appears from the 26th propoſition, that if a body of any ſize was electrified in the ſame degree as the plate Bd, and a communication was made between that body and the ground, by a canal of the ſame length, breadth and thickneſs as NRS; that then the fluid in that canal would be impelled with the ſame force as that in NRS, ſuppoſing the fluid in both canals to be incompreſſible; and conſequently, as the quantity of fluid to be moved, and the reſiſtance to its motion is the ſame in both canals, the fluid ſhould move with the ſame rapidity in both: and I ſee no reaſon to think that the caſe will be different, if the communication is made by canals of real fluid.

Therefore what was ſaid in the beginning of this ſection, namely, that as great a ſhock would be produced by making a communication between the conductor and the ground, as between the two ſides of the Leyden vial, by canals of the ſame length and ſame kind, ſeems a neceſſary conſequence of this theory; as the quantity of fluid which paſſes through the canal is, by the ſuppoſition, the ſame in both; and there is the greateſt reaſon to think, that the rapidity with which it paſſes will be nearly if not quite the ſame in both. I hope ſoon to be able to ſay whether this agrees with experiment as well as theory.

It may be worth obſerving, that the longer the canal N R S is, by which the communication is made, the leſs will be the rapidity with which the fluid moves along it; for the longer the canal is, the greater is the reſiſtance to the motion of the fluid in it; whereas the force with which the whole quantity of fluid in it is impelled, is the ſame whatever be the length of the canal. Accordingly, it is found in melting ſmall wires, by directing a ſhock through them, that the longer the wire the greater charge it requires to melt it.

As the fluid in Bδ, is attracted with great force by the redundant matter in E φ, it is plain that if the fluid is able to penetrate at all into the glaſs, great part of the redundant fluid will be lodged in

b δ,

$b\delta$, and in like manner there will be a great deficience of fluid in $e\varphi$. But in order to form ſome eſtimate of the proportion of the redundant fluid, which will be lodged in $b\delta$, let the communication between Ef and the ground be taken away, as well as that by which Bd is electrified; and let ſo much fluid be taken from $B\delta$, as to make the redundant fluid therein equal to the deficient fluid in $E\varphi$. If we ſuppoſe that all the redundant fluid is collected in $b\delta$, and all the deficient in $e\varphi$, ſo as to leave Bd and Ef ſaturated; then, if the electric repulſion is inverſely as the ſquare of the diſtance, a particle of fluid placed anywhere in the plane bd, except near the extremities b and d, will be attracted with very near as much force by the redundant matter in $e\varphi$, as it is repelled by the redundant fluid in $b\delta$; but if the repulſion is inverſely, as ſome higher power than the ſquare, it will be repelled with much more force by $b\delta$, than it is attracted by $e\varphi$, provided the depth $b\beta$ is very ſmall in reſpect of the thickneſs of the glaſs; and if the repulſion is inverſely, as ſome lower power than the ſquare, it will be attracted with much more force by $e\varphi$, than it is repelled by $b\delta$. Hence it follows, that if the depth to which the fluid can penetrate is very ſmall in reſpect of the thickneſs of the glaſs, but yet is ſuch that the quantity of fluid naturally contained in $b\delta$, or $e\varphi$, is conſiderably more than the redundant fluid in $B\delta$; then, if the repulſion is inverſely as the ſquare of the diſtance, almoſt all the redundant fluid will be collected in $b\delta$, leaving the plate Bd not very much overcharged; and in like manner Ef will be not very much undercharged: if the repulſion is inverſely as ſome higher power than the ſquare, B*d* will be very much overcharged, and E*f* very much undercharged: and if the repulſion is inverſely, as ſome lower power than the ſquare, B*d* will be very much undercharged, and E*f* very much overcharged.

Suppoſe, now, the plate B*d* to be ſeparated from the plate of glaſs, ſtill keeping it parallel thereto, and oppoſite to the ſame part of it that it before was applied to; and let the repulſion of the particles be inverſely, as ſome higher power of the diſtance than the ſquare. When the plate is in contact with the glaſs, the repulſion of the redundant fluid in that plate, on a particle in the plane *b d*, *id eſt*, the inner ſurface of the plate, muſt be equal to the exceſs of the repulſion of the redundant fluid in *b δ* on it, above the attraction of E*φ* on it; therefore, when the plate B*d* is removed ever ſo ſmall a diſtance from the glaſs, the repulſion of the redundant fluid in the plate, on a particle in the inner ſurface of that plate, will be greater than the exceſs of the repulſion of *b δ* on it, above the attraction of E*φ*; for the repulſion of *b δ* will be much more diminiſhed by the removal, than the attraction of E*φ*: conſequently, ſome fluid will fly from the plate to the glaſs, in the form of ſparks: ſo that the plate will not be ſo much overcharged when removed from the glaſs, as it was when in contact with it. I ſhould imagine, however, that it would ſtill be conſiderably overcharged.

If one part of the plate is ſeparated from the glaſs before the reſt, as muſt neceſſarily be the caſe, if it conſiſts of bending materials, I ſhould gueſs it would be at leaſt as much, if not more, overcharged, when ſeparated, as if it is ſeparated all at once.

In like manner, it ſhould ſeem that the plate E*f* will be conſiderably undercharged, when ſeparated from the glaſs, but not ſo much ſo as when in contact with it.

From the ſame kind of reaſoning I conclude, that if the repulſion is inverſely, as ſome lower power of the diſtance than the ſquare, the plate B*d* will be conſiderably undercharged, and E*f* conſiderably overcharged, when ſeparated from the glaſs, but not in ſo great a degree as when they are in contact with it.

§ 7. There is an experiment of Mr. Wilke and Æpinus, related by Dr. Prieſtly, p. 258. called by them, electrifying a plate of air: it conſiſted in placing two large boards of wood, covered with tin plates, parallel to each other, and at ſome inches aſunder. If a communication was made between one of theſe and the ground, and the other was poſitively electrified, the former was undercharged; the boards ſtrongly attracted each other; and, on making a communication between them, a ſhock was felt like that of the Leyden vial.

I am uncertain whether in this experiment the air contained between the two boards is very much overcharged on one ſide, and very much undercharged on the other, as is the caſe with the plate of glaſs in the Leyden vial; or whether the caſe is, that the redundant or deficient fluid is lodged only in the two boards, and that the air between them ſerves only to prevent the electricity from running from one board to the other: but whichever of theſe is the caſe, the experiment is equally conformable to the theory.

It muſt be obſerved, that a particle of fluid placed between the two plates is drawn towards the undercharged plate, with a force exceeding that with which it would be repelled from the overcharged plate, if it was electrified with the ſame force, the other plate being taken away, nearly in the ratio of twice the quantity of redundant fluid actually contained in the plate, to that which it would contain, if electrified with the ſame force by itſelf; ſo that, unleſs the plate is very weakly electrified, or their diſtance is very conſiderable, the fluid will be apt to fly from one to the other, in the form of ſparks.

§ 8. Whenever any conducting body as A, communicating with the ground, is brought ſufficiently near an overcharged body B, the electric fluid is apt to fly through the air from B to A, in the form of a ſpark: the way by which this is brought about ſeems to be this. The fluid placed anywhere between the two bodies, is repelled from B towards A, and will conſequently move ſlowly through the air from one to the other: now it ſeems as if this motion increaſed the elaſticity of the air, and made it rarer: this will enable the fluid to flow in a ſwifter current, which will ſtill further increaſe the elaſticity of the air, till at laſt it is ſo much rarified, as to form very little oppoſition to the motion of the electric fluid, upon which it flies in an uninterrupted maſs from one body to the other.

In the ſame manner may the electric fluid paſs from one body to another, in the form of a ſpark, if the firſt body communicates with the ground, and the other body is negatively electrified, or in any other caſe in which one body is ſtrongly diſpoſed to part with its electricity to the air, and the other is ſtrongly diſpoſed to receive it.

In like manner, when the electric fluid is made to paſs through water, in the form of a ſpark, as in Signor Beccaria's * and Mr. Lane's † experiments, I imagine that the water, by the rapid motion of the electric fluid through it, is turned into an elaſtic fluid, and ſo much rarified as to make very little oppoſition to its motion: and when ſtones are burſt or thrown out from buildings ſtruck by lightning, in all probability that effect is cauſed by the moiſture in the ſtone, or ſome of the ſtone itſelf, being turned into an elaſtic fluid.

It appears plainly, from the ſudden riſing of the water in Mr. Kinnerſley's electrical air thermometer ‡, that when the electric fluid paſſes through the air, in the form of a ſpark, the air in its paſſage is either very much rarified, or intirely diſplaced: and the burſting of the glaſs veſſels, in Beccaria's and Lane's experiments, ſhews that the ſame thing happens with regard to the water, when the electric fluid paſſes through it in the form of a ſpark. Now, I ſee no means by which the diſplacing of the air or

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* Elettriciſmo artificiale e naturale, p. 110. Prieſtly, p. 209.

† Phil. Tranſ. 1767, p. 451.

‡ Phil. Tranſ. 1763, p. 84. Prieſtly, p. 216.

water can be brought about, but by ſuppoſing its elaſticity to be increaſed, by the motion of the electric fluid through it, unleſs you ſuppoſe it to be actually puſhed aſide, by the force with which the electric fluid endeavours to iſſue from the over-charged body: but I can by no means think, that the force with which the fluid endeavours to iſſue, in the ordinary caſes in which electric ſparks are produced, is ſufficient to overcome the preſſure of the atmoſphere, much leſs that it is ſufficient to burſt the glaſs veſſels in Beccaria's and Lane's experiments.

The truth of this is confirmed by Prop. XVI. For, let an undercharged body be brought near to, and oppoſite to the end of a long cylindrical body communicating with the ground, by that propoſition the preſſure of the electric fluid againſt the baſe of the cylinder is ſcarcely greater than the force with which the two bodies attract each other, provided that no part of the cylinder is undercharged; which is very unlikely to be the caſe, if the electric repulſion is inverſely as the ſquare of the diſtance, as I have great reaſon to believe it is; and, conſequently, if the ſpark was produced, by the air being puſhed aſide by the force with which the fluid endeavours to iſſue from the cylinder, no ſparks ſhould be produced, unleſs the electricity was ſo ſtrong, that the force with which the bodies attracted each other was as great as the preſſure of the atmoſphere againſt the baſe of the cylinder: whereas it is well known, that a ſpark may be produced, when the force, with which the bodies attract, is very trifling in reſpect of that.

One may frequently obſerve, in diſcharging a Leyden vial, that if the two knobs are approached together very ſlowly, a hiſſing noiſe will be perceived before the ſpark; which ſhews, that the fluid begins to flow from one knob to the other, before it paſſes in the form of a ſpark; and therefore ſerves to confirm the truth of the opinion, that the ſpark is brought about in the gradual manner here deſcribed.

Fig. 1.

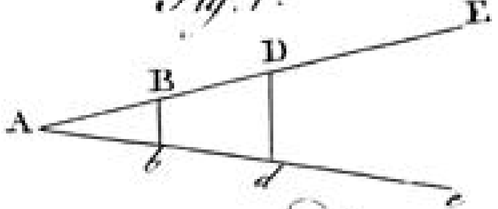


Fig. 2.

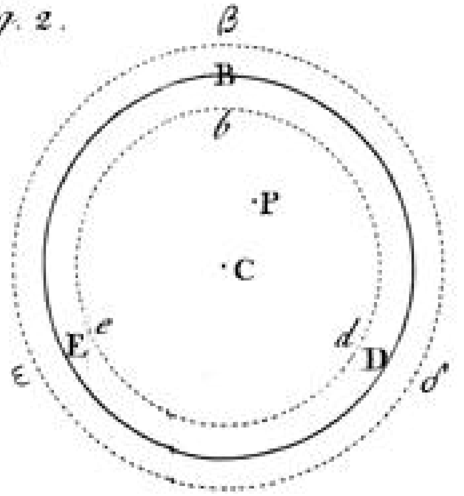
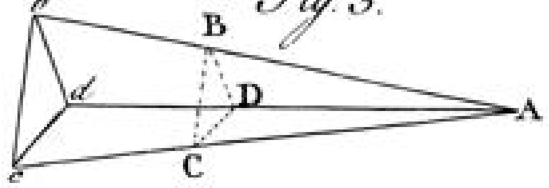


Fig. 3.



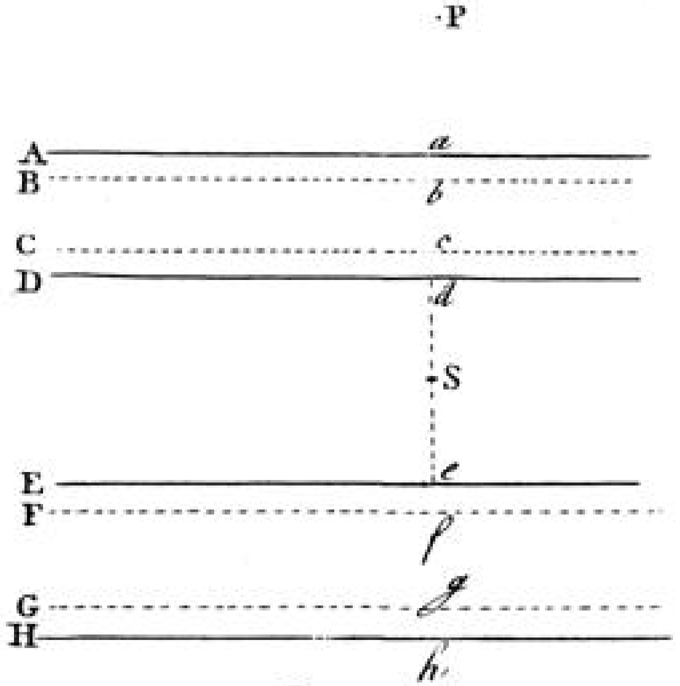
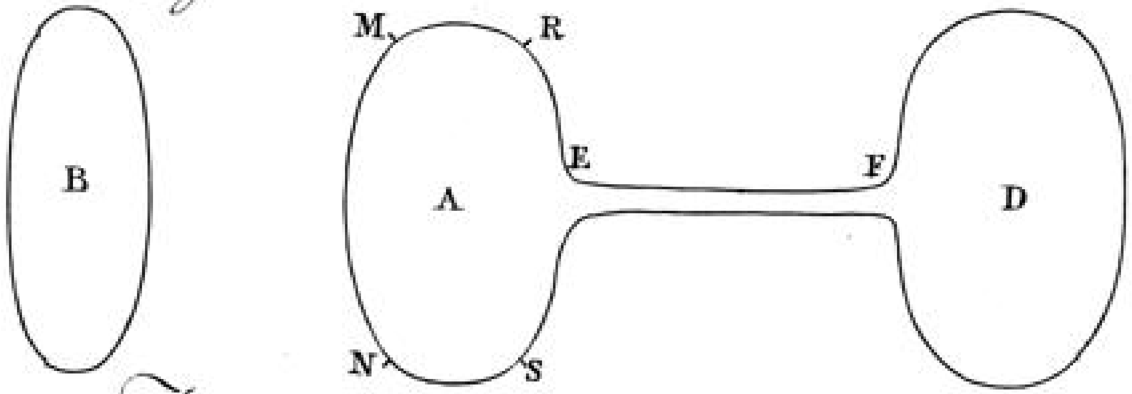


Fig. 4.

Fig. 5.



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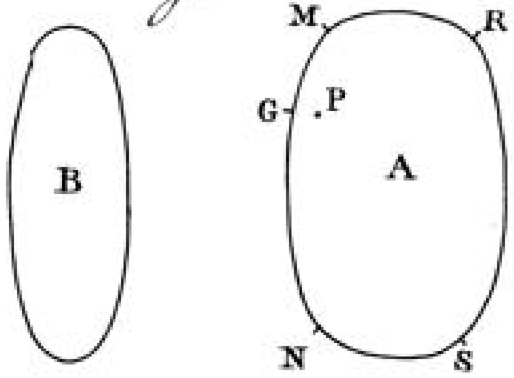
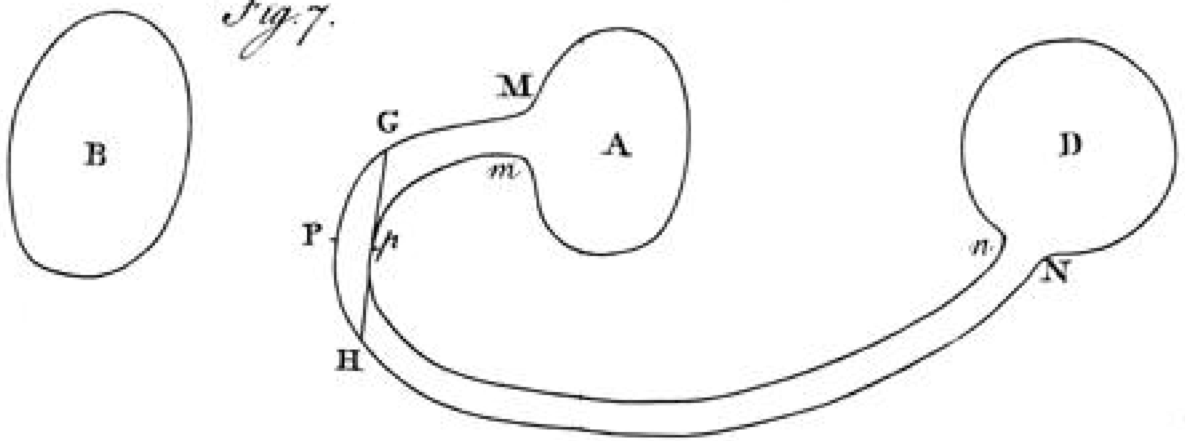


Fig. 7.



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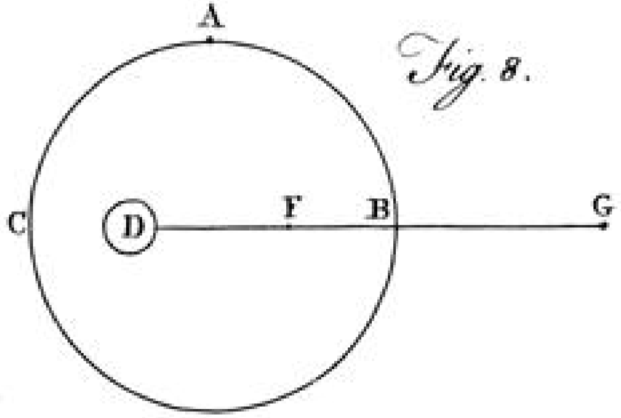

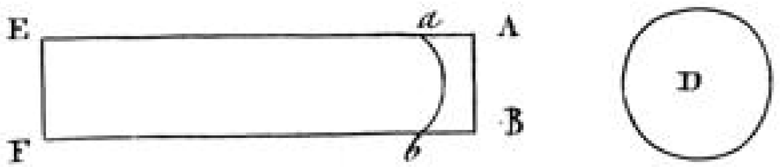

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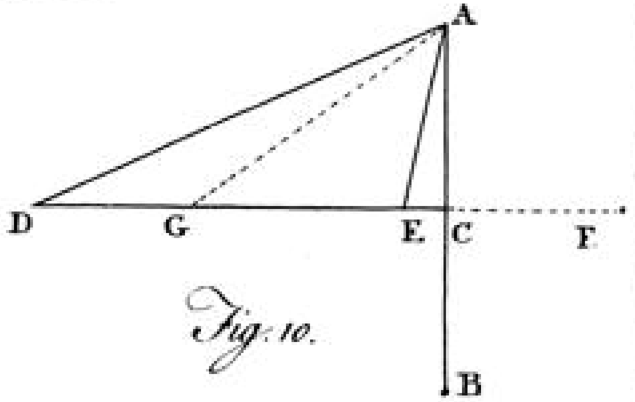

Fig. 10.

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Philos. Trans. Vol. LXI. Tab XXI. p.623.

Fig. 11.

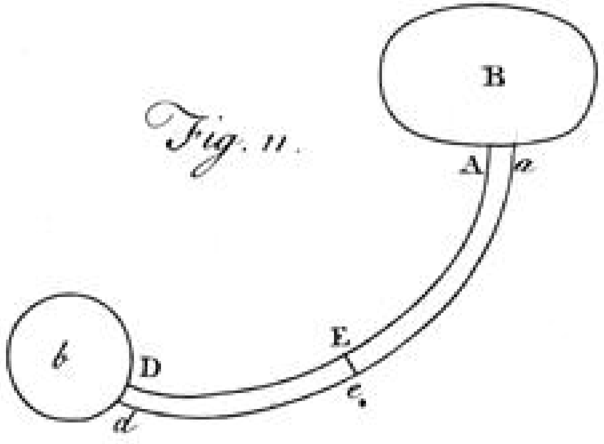


Fig. 12.

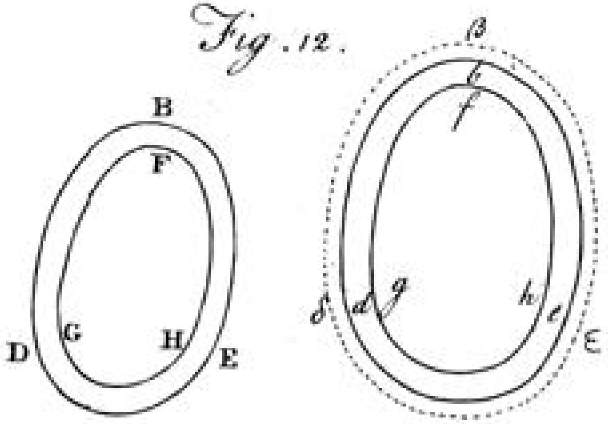
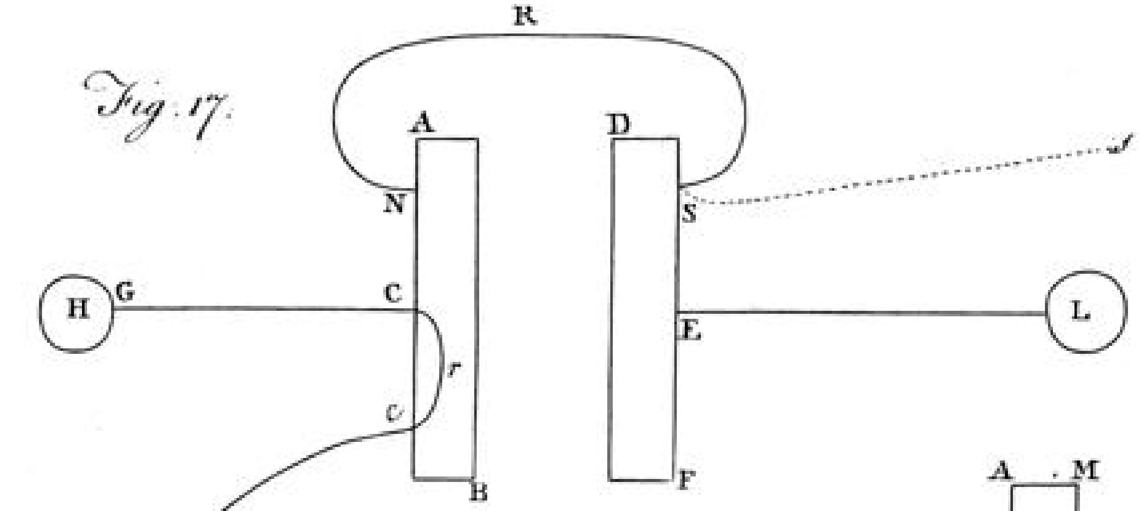


Fig. 17.



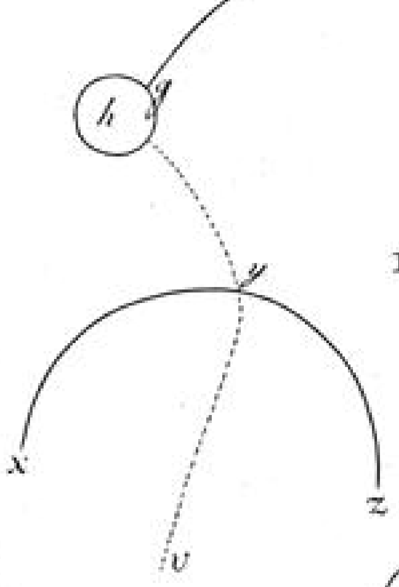


Fig. 13.

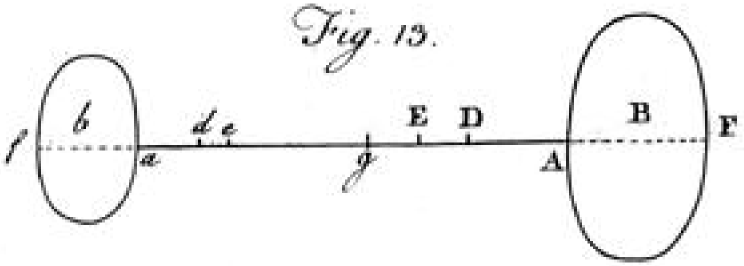
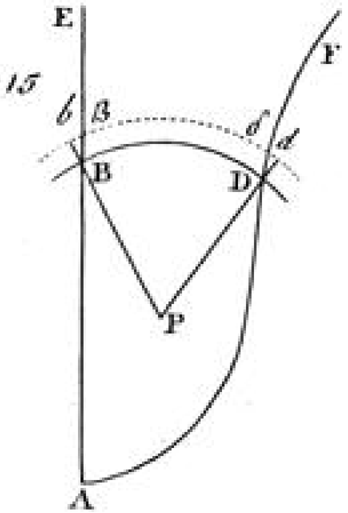


Fig. 15.



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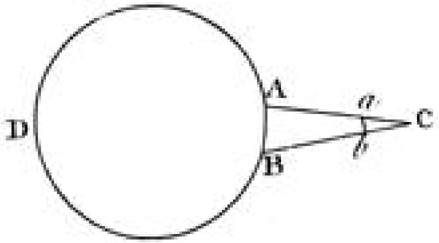

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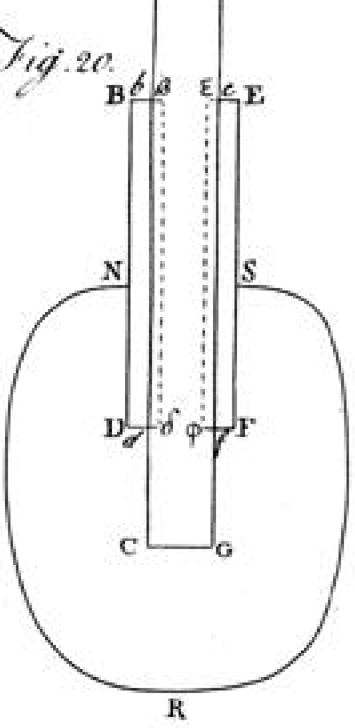

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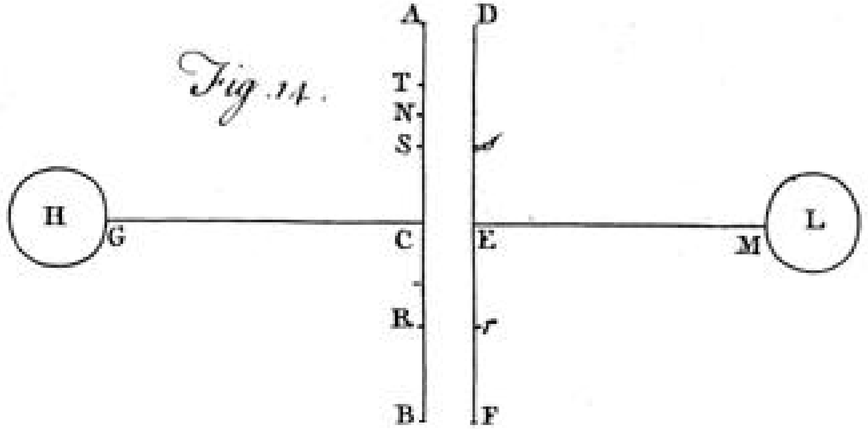

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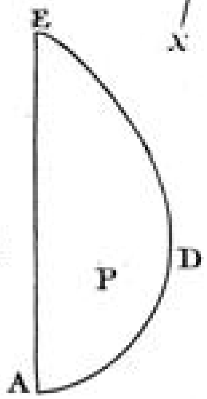

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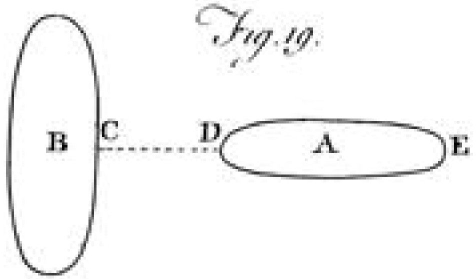

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